A Multi-Objective Sub-Domain Optimization Algorithm for Sensor Placement in Water Distribution Systems

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Background:

- Battle of Water Sensor Network (BWSN) session hosted by the University of Cincinnati (2006).

- Two water distribution systems were analyzed to place a finite number of sensors at a large number of potential locations.

- Four criteria for the evaluation of the performance of the design.
  - Min. Expected time of detection: $Z_1$
  - Min. Expected population affected prior to detection: $Z_2$
  - Min. Expected volume of contaminated water prior to detection: $Z_3$
  - Max. Detection likelihood or reliability: $Z_4$
Background:

- Several Algorithms were proposed:
  - Optimization based on single objective(s)
  - Optimization based on multi-objectives

- Solutions procedures used were also varied:
  - Enumeration
  - Integer programming
  - Heuristic methods
  - Pareto front analysis (Non-dominated Sorting Algorithms).
Background:

- Some CHALLENGES:
  - Computational efficiency;
  - Computational memory requirements;
  - Applicability to the solution of large scale systems; and,
  - The question of how to handle the non-detects in this analysis?

\[\text{Sub-domain method}\]

\[\text{Effects?}\]
Mathematical Model:

- Multi-objective optimization model:

\[ f_1 = \text{minimize} \left\{ \frac{1}{N_s} \sum_{s=1}^{N_s} t^s_d(X) \right\} \]

\[ f_2 = \text{minimize} \left\{ \frac{1}{N_s} \sum_{s=1}^{N_s} \sum_{i=1}^{N} \sum_{t=t^n}^{t} V^s_i \right\} \]

\[ f_3 = \text{maximize} \left\{ \frac{1}{N_s} \sum_{s=1}^{N_s} d^s(X) \right\} \]

subject to \( \sum x_i = M \)

where

\( X = [x_1, x_2, \ldots, x_n]^T \)

\( x_i = \{0, 1\} \)

\( M: \) given number of sensors.

\( n: \# \) candidate junctions.

\( N: \# \) of junctions in WDS.

\( N_s: \# \) of scenarios.

\( V_i(t): \) Contaminated water Vol.

\( d^s: \{0, 1\} \) detection parameter
Sub-domain Solution Algorithm:

- Total combinations of $n$ junctions and $k$ sensors:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- Examples
  - Candidate junctions = 129
  - Sensor = 5
  - Total combinations = 275,234,400
  - Candidate junctions = 30
  - Sensor = 5
  - Total combinations = 142,506

$$\text{Ratio} = \frac{142,506}{275,234,400} = 0.05\%$$
Sub-domain Solution Algorithm:

- Flowchart

1. Generate contaminant scenarios
2. Run EPANET to obtain data needed in optimization for all scenarios
3. Select initial sub-domain by roulette wheel
4. Generate initial population
5. Find Pareto optimal front using NSGA-II within sub-domain
6. Get all junctions and solutions in Pareto optimal front
7. Stop if stopping iteration condition is satisfied; otherwise, update the sub-domain and repeat the process.

Flowchart:

- Generate contaminant scenarios
- Run EPANET to obtain data needed in optimization for all scenarios
- Select initial sub-domain by roulette wheel
- Generate initial population
- Find Pareto optimal front using NSGA-II within sub-domain
- Get all junctions and solutions in Pareto optimal front
- Stop if stopping iteration condition is satisfied; otherwise, update the sub-domain.
Sub-domain Solution Algorithm:

- Remarks

  - Initial junctions are selected based on order of importance and roulette wheel method.

  - Non-dominated sorting genetic algorithm (NSGA-II) works within reduced subsets of junctions (sub-domain).

  - All junctions in Pareto optimal front directly enter the sub-domain in next iteration.

  - Additional junctions will fill the remaining slots based on the order of importance at some probability using the roulette wheel method.

  - Maximum iterations or same Pareto optimal fronts obtained in two consecutive iterations is chosen as stopping criterion.

  - All junctions are cycled through the sub-domain based on some probability.
Numerical Applications:

- Water Distribution System 1
  - 129 junctions
  - 169 pipes
  - A reservoir
  - 2 storage tanks
  - 2 pumping stations
Numerical Applications:

- **Water Distribution System 1**
  - **Parameters:**
    - Five sensors are placed.
    - Scenarios in optimization: 20 contamination scenarios for each junction resulting in total scenarios of 2580.
    - 30 candidate junctions in each sub-domain.
    - Maximum number of iterations is chosen as 20.
    - Scenarios in evaluation: Same number scenarios but generated independently.
    - Evaluation of the outcome is done by an independent software used in BWSN 2006.

- Time of detection for non-detected scenarios is set as length of duration as a penalty.
Numerical Applications: (ND included in optimization)

- Water Distribution System 1
  - Case 1: Three objectives are selected in optimization, none detects are considered
Numerical Applications: (ND Excluded in evaluation)

- **Water Distribution System 1**
  - **Case 1:** Three objectives are selected in optimization

<table>
<thead>
<tr>
<th>Solution</th>
<th>Junction ID</th>
<th>$Z_1$ (minutes)</th>
<th>$Z_2$ (Gal)</th>
<th>$Z_3$ (Gal)</th>
<th>$Z_4$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>J10, J45, J83, J100, J126</td>
<td>1,263.4</td>
<td>672.5</td>
<td>43,065.1</td>
<td>83.80</td>
</tr>
<tr>
<td>2</td>
<td>J10, J45, J83, J100, J118</td>
<td>1,050.1</td>
<td>388.3</td>
<td>12,879.6</td>
<td>83.02</td>
</tr>
<tr>
<td>3</td>
<td>J10, J68, J83, J118, J122</td>
<td>919.1</td>
<td>301.8</td>
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<tr>
<td>4</td>
<td>J17, J49, J68, J83, J102</td>
<td>715.5</td>
<td>148.3</td>
<td>2,780.6</td>
<td>70.16</td>
</tr>
</tbody>
</table>

where

- $Z_1$: expected time of detection,
- $Z_2$: expected population affected
- $Z_3$: expected volume contaminated,
- $Z_4$: detection likelihood

The results shown are for the 100$^{th}$ iteration.
Numerical Applications:

- Water Distribution System 1
  - The same as Case 1 three objectives are selected.
  - The purpose is to analyze the effect of the non-detected scenarios on the Pareto optimal front.
  - This application is identified as (Case 3).
Numerical Applications:
Numerical Applications: (ND excluded in calc. & eval.)

- Water Distribution System 1

- The previous problem has been solved by Preis and Ostfeld, 2006.

- They have used NSGA applied to the whole domain.

- In that study the minimization of the expected time of detection and maximization of detection likelihood were selected as design objectives.

- The non-detected scenarios were excluded in calculation of time of detection.

- This is a good test case to demonstrate the effectiveness and efficiency of the proposed algorithm and also the effect of non-detects on the solution.

(Case 2)
Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 1**
  - **Case 2**: Minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.
  - The non-detected scenarios are excluded in calculation of time of detection for testing the effectiveness and efficiency of the algorithm.

![Graph showing the relationship between time of detection (minute) and detection likelihood (%)]
Numerical Applications: (ND excluded in calc. & eval.)

- Water Distribution System 1
  
  **Case 2:** minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.

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<tr>
<td>3</td>
<td>J21, J68, J83, J99, J118</td>
<td>597.4</td>
<td>190.2</td>
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<td>4</td>
<td>J21, J31, J43, J58, J93</td>
<td>394.6</td>
<td>138.1</td>
<td>6,649.3</td>
<td>26.55</td>
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Solutions obtained by NSGA-II in Preis and Osfeld, 2006

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<tr>
<td>1</td>
<td>J29, J30, J34, J43, J49</td>
<td>517.9</td>
<td>212.6</td>
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<tr>
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<td>J21, J46, J68, J101, J116</td>
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<td>154.8</td>
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<td>682.1</td>
<td>241.6</td>
<td>8,165.2</td>
<td>66.04</td>
</tr>
</tbody>
</table>

Solutions reported by NSGA-II in Preis and Osfeld, 2006 after 440 generations
In this solution the reported results are obtained after 70 generations (3 iter/sub).
Numerical Applications:

- Water Distribution System 2
  - 12,523 junctions
  - 14,822 pipes
  - 2 reservoirs
  - 2 storage tanks
  - 4 pumping stations
Numerical Applications:

- **Water Distribution System 2**
  - **Parameters**
    - Five sensors are placed
    - Scenarios in optimization: 3000 contamination scenarios are randomly generated
    - Scenarios in evaluation: same number scenarios but generated independently
    - 100 candidate junctions in each sub-domain
    - Maximum iterations is given as 50
    - Evaluation software used in BWSN 2006
Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 2**
  - Minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.
  - The non-detected scenarios are excluded in calculation of time of detection for testing the effectiveness and efficiency of the algorithm.
### Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 2**
  - Minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.

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<th>$Z_4$ (%)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>J1486, J3747, J4247, J8452, J10874</td>
<td>1,055.2</td>
<td>2,338.1</td>
<td>206,837.3</td>
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<td>2</td>
<td>J1486, J3301, J4247, J4684, J10393</td>
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<td>1,865.0</td>
<td>122,546.8</td>
<td>17.33</td>
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<td>J32, J4247, J4562, J4771, J13349</td>
<td>427.5</td>
<td>1,083.2</td>
<td>48,201.9</td>
<td>4.60</td>
</tr>
</tbody>
</table>

Solutions obtained by NSGA-II in Preis and Osfeld, 2006
Conclusions:

- The multi-objective sub-domain optimization model proposed can be effectively used in the solution of design of water sensor network in large water distribution systems.

- The algorithm, based on NSGA-II and sub-domain, is an effective approach for solving multi-objective optimization model.

- Inclusion of non-detects into analysis algorithmically excludes most solutions on the Pareto front which may not be desirable solutions.

- Non-detected scenarios can be included in the calculation of objectives in both design and evaluation phases. Simulation duration time can be used for this as a penalty function to include the impact of the non-detected scenarios.
Thank You.

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