A Multi-Objective Sub-Domain Optimization Algorithm for Sensor Placement in Water Distribution Systems

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Background:

- **Battle of Water Sensor Network (BWSN) session hosted by the University of Cincinnati (2006).**

- **Two water distribution systems were analyzed to place a finite number of sensors at a large number of potential locations.**

- **Four criteria for the evaluation of the performance of the design.**

  - **Min. Expected time of detection: Z_1**

  - **Min. Expected population affected prior to detection: Z_2**

  - **Min. Expected volume of contaminated water prior to detection: Z_3**

  - **Max. Detection likelihood or reliability: Z_4**

Background:

- **Several Algorithms were proposed:**
  - Optimization based on single objective(s)
  - Optimization based on multi-objectives

- **Solutions procedures used were also varied:**
  - Enumeration
  - Integer programming
  - Heuristic methods
  - Pareto front analysis (Non-dominated Sorting Algorithms).

Background:

- **Some CHALLENGES:**
  - **Computational efficiency;**
  - **Computational memory requirements;**
  - **Applicability to the solution of large scale systems; and,**

  } Sub-domain method

  - **The question of how to handle the non-detects in this analysis?**

  } Effects?

Mathematical Model:

- **Multi-objective optimization model:**

$$f_1 = \text{minimize}\left\{\frac{1}{N_s}\sum_{s=1}^{N_s} t_d^s(X)\right\}$$

$$f_2 = \text{minimize}\left\{\frac{1}{N_s}\sum_{s=1}^{N_s}\sum_{i=1}^{N}\sum_{t=t_s^{in}}^{t_d^s(X)} V_i^s\right\}$$

$$f_3 = \text{maximize}\left\{\frac{1}{N_s}\sum_{s=1}^{N_s} d_s(X)\right\}$$

$$\textit{subject to} \quad \sum x_i = M$$

where

$X=[x_1, x_2, \ldots, x_n]^T$

$x_i = \{0, 1\}$

M: given number of sensors.

n: # candidate junctions.

N: # of junctions in WDS.

N_s : # of scenarios.

$V_i(t)$: Contaminated water Vol.

d_s: {0, 1} detection parameter

Sub-domain Solution Algorithm:

- **Total combinations of *n* junctions and *k* sensors:**

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- **Examples**

  Candidate junctions = 129

  Sensor = 5

  Total combinations = 275,234,400

  Candidate junctions = 30

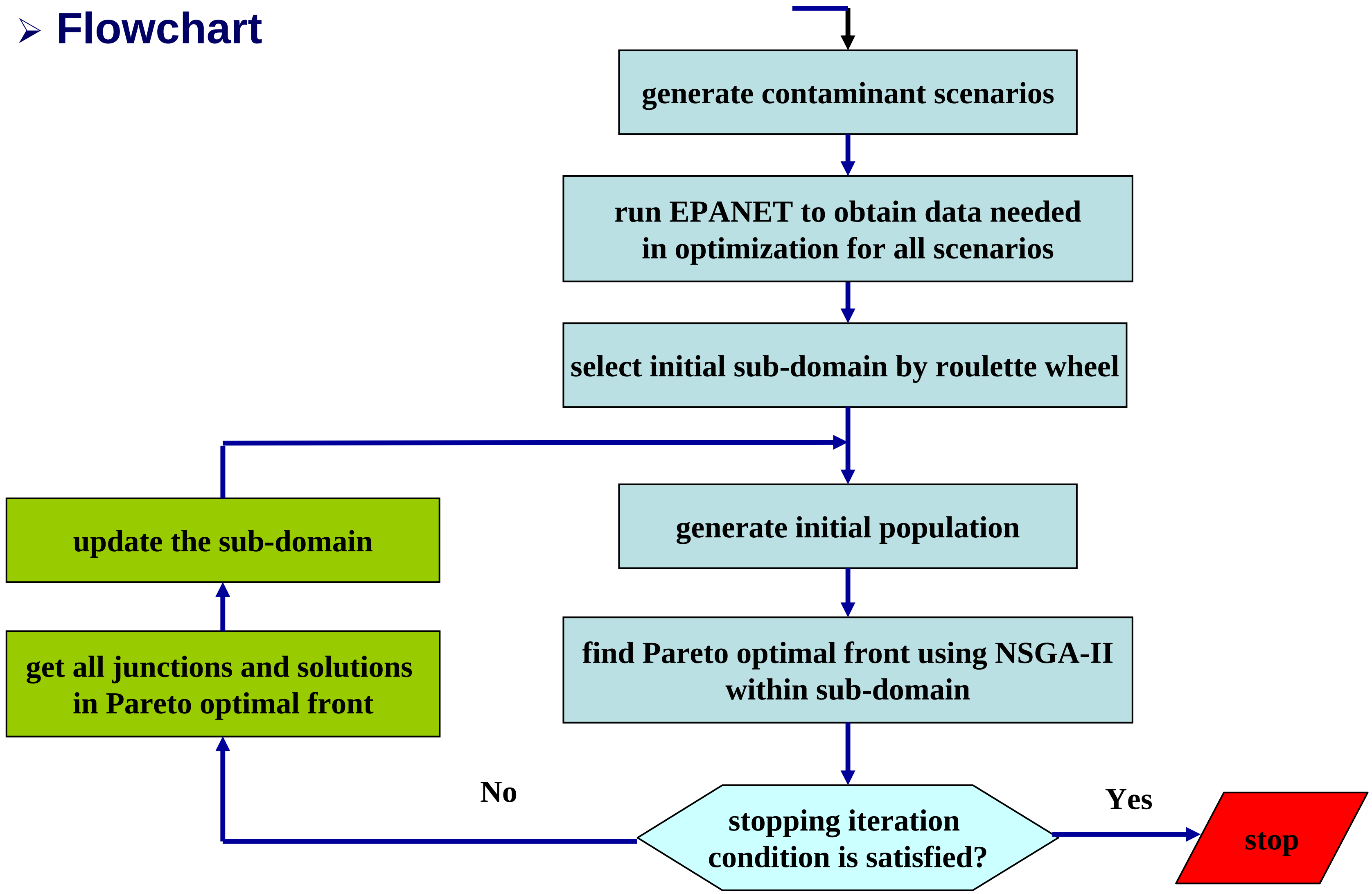
  Sensor = 5

  Total combinations = 142,506

$$Ratio = \frac{142,506}{275,234,400} = 0.05\%$$

Sub-domain Solution Algorithm:

- Flowchart

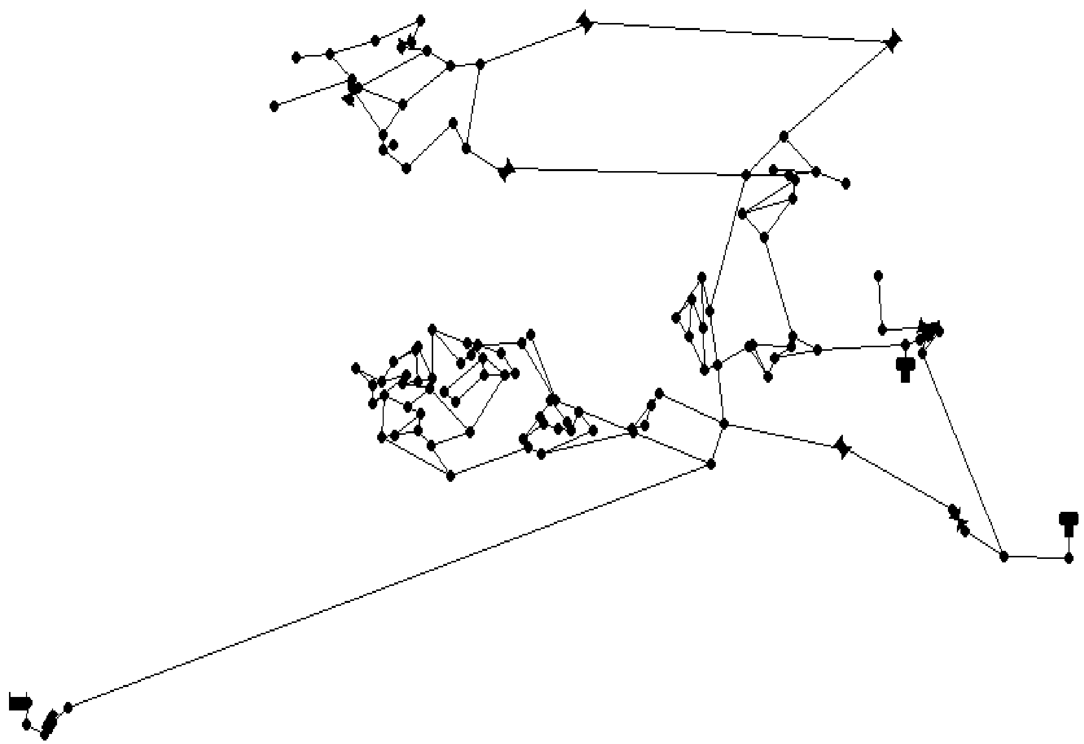


Sub-domain Solution Algorithm:

- **Remarks**
  - **Initial junctions are selected based on order of importance and roulette wheel method.**
  - **Non-dominated sorting genetic algorithm (NSGA-II) works within reduced subsets of junctions (sub-domain).**
  - **All junctions in Pareto optimal front directly enter the sub-domain in next iteration.**
  - **Additional junctions will fill the remaining slots based on the order of importance at some probability using the roulette wheel method.**
  - **Maximum iterations or same Pareto optimal fronts obtained in two consecutive iterations is chosen as stopping criterion.**
  - **All junctions are cycled through the sub-domain based on some probability.**

Numerical Applications:

- Water Distribution System 1



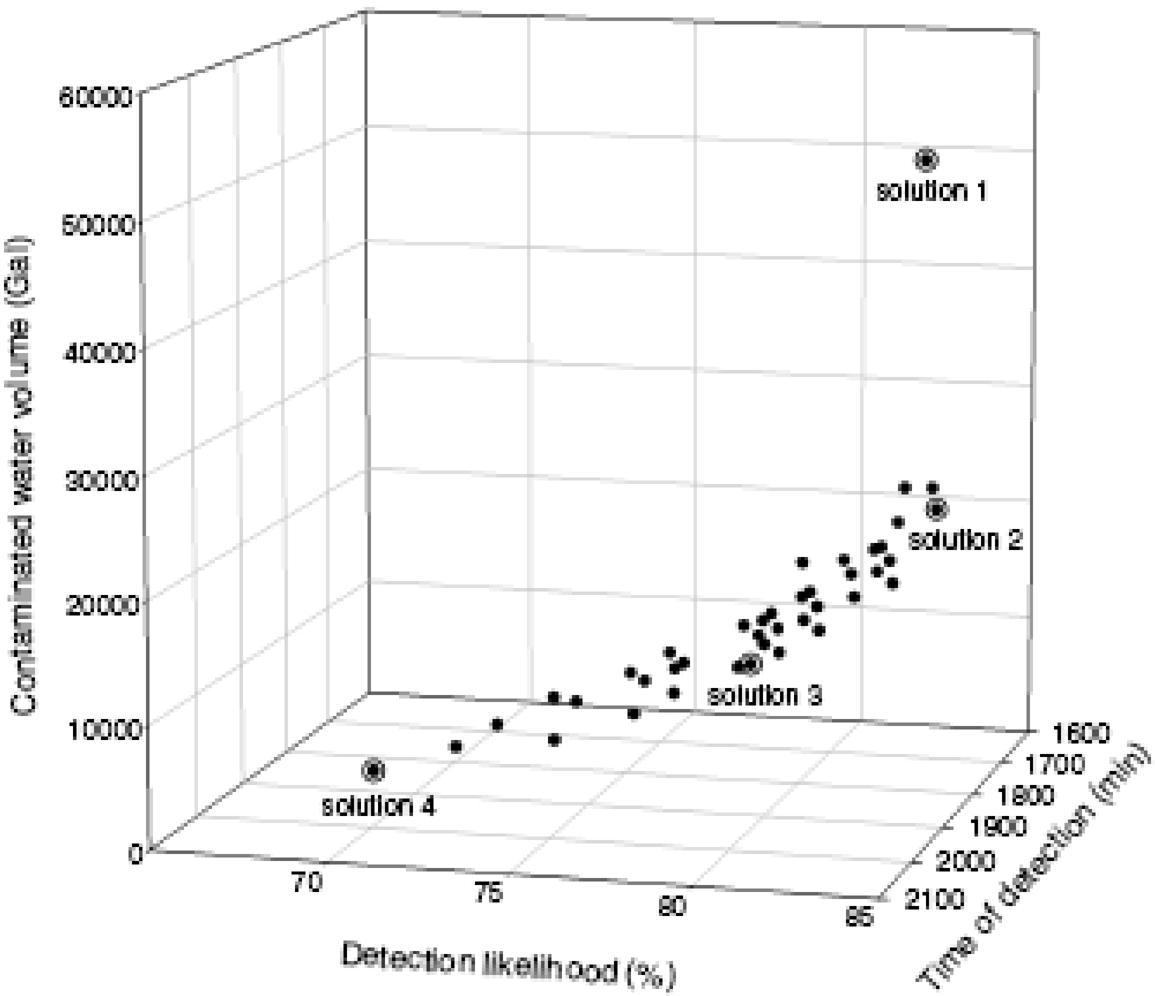
- 129 junctions
- 169 pipes
- A reservoir
- 2 storage tanks
- 2 pumping stations

Numerical Applications:

- Water Distribution System 1
  - Parameters:
    - Five sensors are placed.
    - Scenarios in optimization: 20 contamination scenarios for each junction resulting in total scenarios of 2580.
    - 30 candidate junctions in each sub-domain.
    - Maximum number of iterations is chosen as 20.
    - Scenarios in evaluation: Same number scenarios but generated independently.
    - Evaluation of the outcome is done by an independent software used in BWSN 2006.
    - Time of detection for non-detected scenarios is set as length of duration as a penalty.

Numerical Applications: (ND included in optimization)

- **Water Distribution System 1**
  - **Case 1:** Three objectives are selected in optimization, none detects are considered.



Numerical Applications: (ND Excluded in evaluation)

- **Water Distribution System 1**
  - **Case 1: Three objectives are selected in optimization**

Solution	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
1	J10, J45, J83, J100, J126	1,263.4	672.5	43,065.1	83.80
2	J10, J45, J83, J100, J118	1,050.1	388.3	12,879.6	83.02
3	J10, J68, J83, J118, J122	919.1	301.8	5,416.5	78.49
4	J17, J49, J68, J83, J102	715.5	148.3	2,780.6	70.16

where

Z_1: expected time of detection,

Z_2: expected population affected

Z_3 : expected volume contaminated,

Z_4: detection likelihood

The results shown are for the 100th iteration.

Numerical Applications:

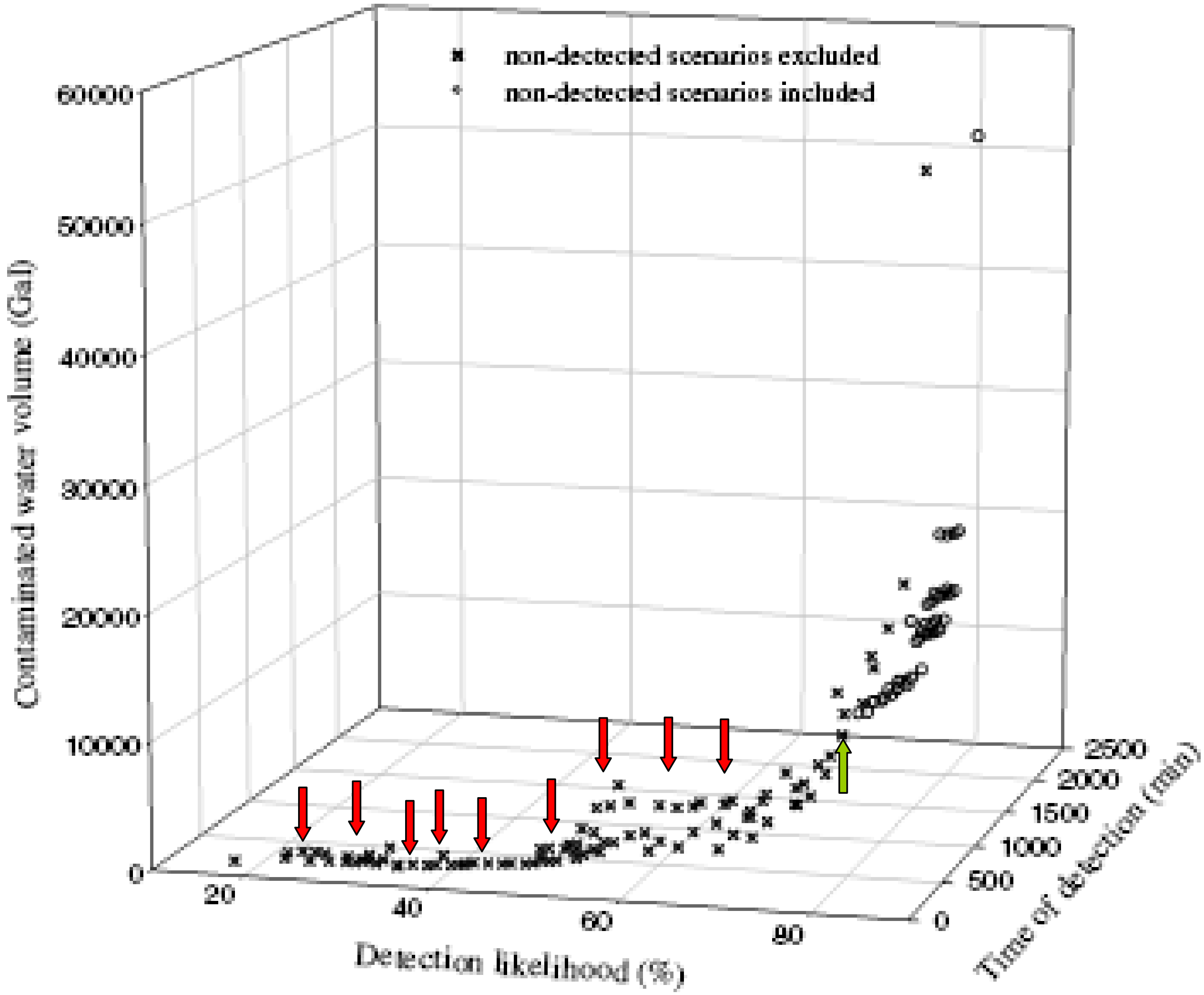
- **Water Distribution System 1**

  - **The same as Case 1 three objectives are selected.**

  - **The purpose is to analyze the effect of the non-detected scenarios on the Pareto optimal front.**

  - **This application is identified as (Case 3).**

Numerical Applications:



Numerical Applications: (ND excluded in calc. & eval.)

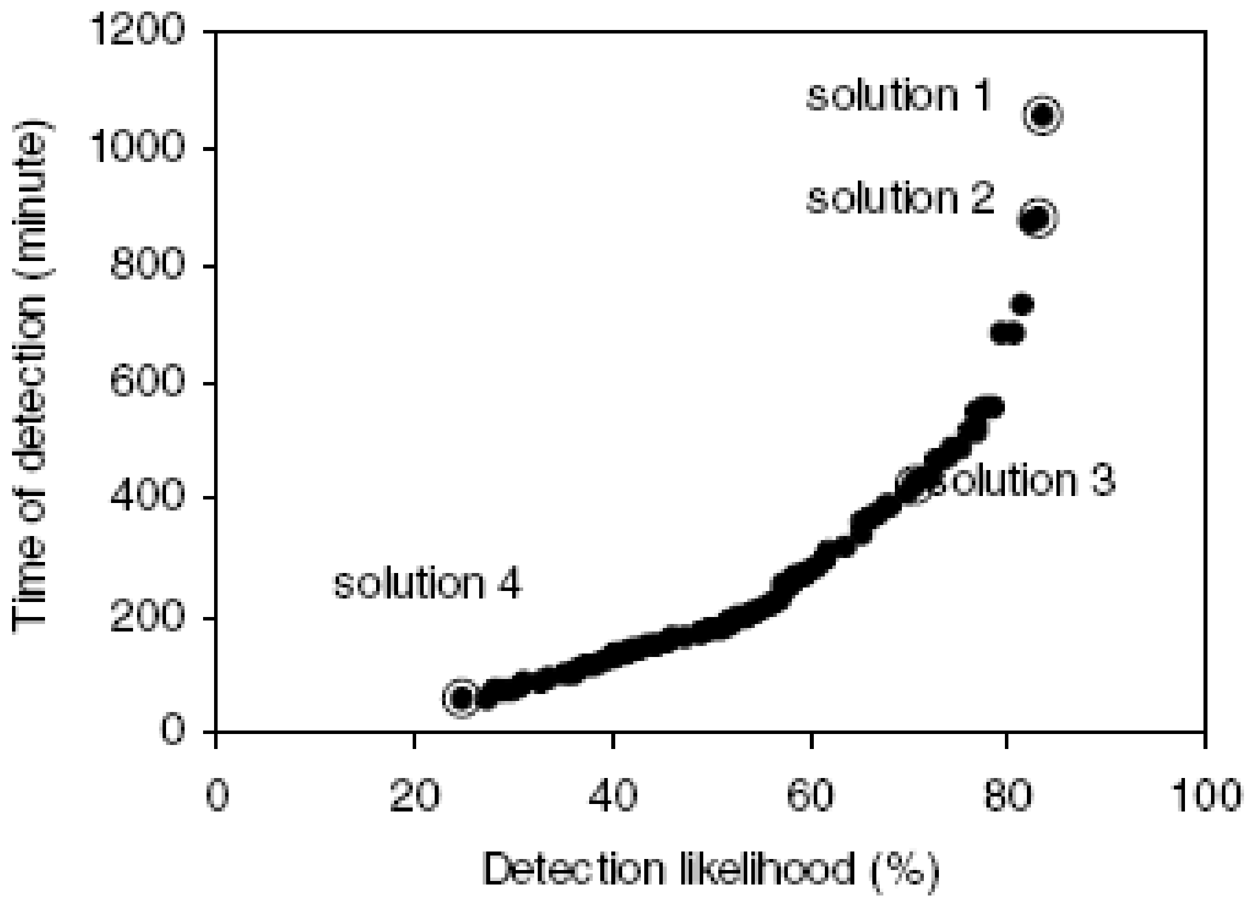
- **Water Distribution System 1**

  - **The previous problem has been solved by Preis and Ostfeld, 2006.**
  - **They have used NSGA applied to the whole domain.**
  - **In that study the minimization of the expected time of detection and maximization of detection likelihood were selected as design objectives.**
  - **The non-detected scenarios were excluded in calculation of time of detection.**
  - **This is a good test case to demonstrate the effectiveness and efficiency of the proposed algorithm and also the effect of non-detects on the solution.**

  (Case 2)

Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 1**
  - **Case 2:** **Minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.**
  - **The non-detected scenarios are excluded in calculation of time of detection for testing the effectiveness and efficiency of the algorithm**



Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 1**
  - **Case 2:** minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.

Solution	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
1	J10, J45, J83, J100, J126	1,263.4	672.5	43,065.1	83.80
2	J10, J45, J83, J100, J118	1,050.1	388.3	12,879.6	83.02
3	J21, J68, J83, J99, J118	597.4	190.2	2,695.5	70.70
4	J21, J31, J43, J58, J93	394.6	138.1	6,649.3	26.55

Solutions obtained by NSGA-II in Preis and Osfeld, 2006

Solution	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
1	J29, J30, J34, J43, J49	517.9	212.6	18,447.1	14.46
2	J21, J46, J68, J101, J116	436.1	154.8	7,106.6	47.56
3	J45, J70, J83, J101, J116	682.1	241.6	8,165.2	66.04

Solutions reported by NSGA-II in Preis and Osfeld, 2006 after 440 generations
In this solution the reported results are obtained after 70 generations (3 iter/sub).

Numerical Applications:

- Water Distribution System 2

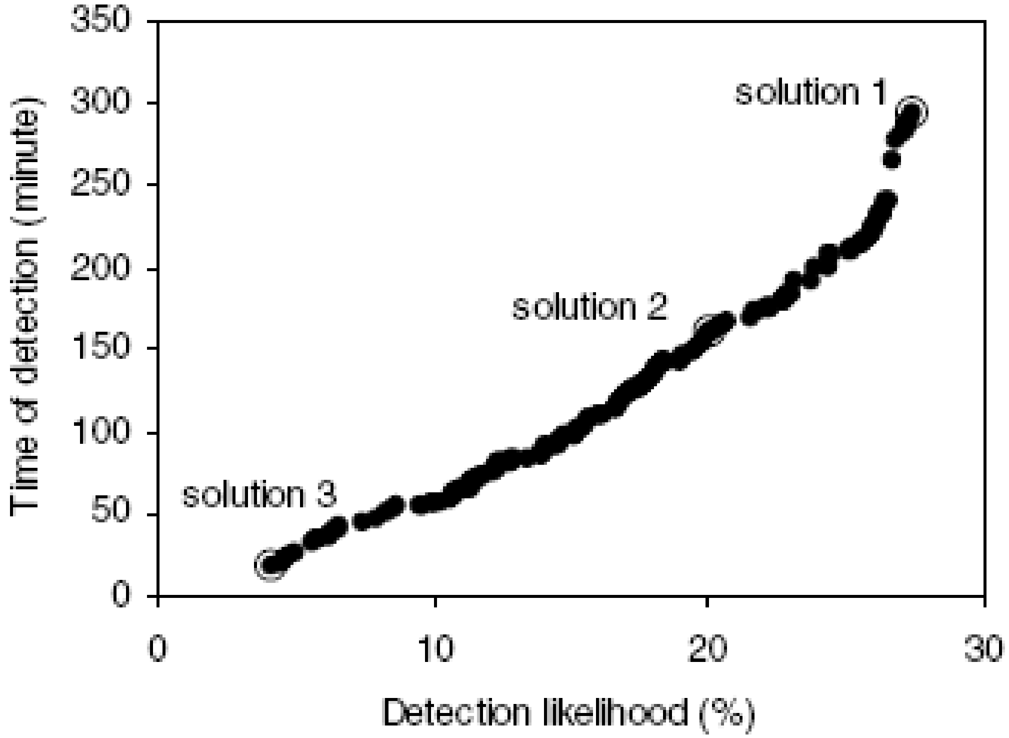
- 12,523 junctions
- 14,822 pipes
- 2 reservoirs
- 2 storage tanks
- 4 pumping stations

Numerical Applications:

- Water Distribution System 2
  - Parameters
    - Five sensors are placed
    - Scenarios in optimization: 3000 contamination scenarios are randomly generated
    - Scenarios in evaluation: same number scenarios but generated independently
    - 100 candidate junctions in each sub-domain
    - Maximum iterations is given as 50
    - Evaluation software used in BWSN 2006

Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 2**
  - **Minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.**
  - **The non-detected scenarios are excluded in calculation of time of detection for testing the effectiveness and efficiency of the algorithm**



Numerical Applications: (ND excluded in calc. & eval.)

- **Water Distribution System 2**
  - **Minimization of the expected time of detection and maximization of detection likelihood are selected as design objectives.**

Solution	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
1	J1486, J3747, J4247, J8452, J10874	1,055.2	2,338.1	206,837.3	28.03
2	J1486, J3301, J4247, J4684, J10393	769.9	1865.0	122,546.8	17.33
3	J32, J4247, J4562, J4771, J13349	427.5	1,083.2	48,201.9	4.60

Solutions obtained by NSGA-II in Preis and Osfeld, 2006

Solution	Junction ID	Z_1 (minutes)	Z_2	Z_3 (Gal)	Z_4 (%)
1	J871, J1917, J2024, J4115, J4247	807.2	1,700.1	122,986.8	16.80
2	J336, J470, J690, J723, J913	522.97	1,486.0	72,446.9	3.03

Conclusions:

- The multi-objective sub-domain optimization model proposed can be effectively used in the solution of design of water sensor network in large water distribution systems.

- The algorithm, based on NSGA-II and sub-domain, is an effective approach for solving multi-objective optimization model.

- Inclusion of non-detects into analysis algorithmically excludes most solutions on the Pareto front which may not be desirable solutions.

- Non-detected scenarios can be included in the calculation of objectives in both design and evaluation phases. Simulation duration time can be used for this as a penalty function to include the impact of the non-detected scenarios.

Thank You.

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