

# **Dynamic Fugacity Approach in Modeling Contaminant Fate and Transport in Rivers**

**EWRI 2007**

**Sinem Gökgöz Kılıç**

**May 17, 2007**

**Multimedia Environmental Simulations Laboratory  
School of Civil and Environmental Engineering  
Georgia Institute of Technology**



# Objective

---

Development of a dynamic fugacity-based model to simulate concentration distribution of a contaminant in the water column as well as sediments of a river reach.



# Outline

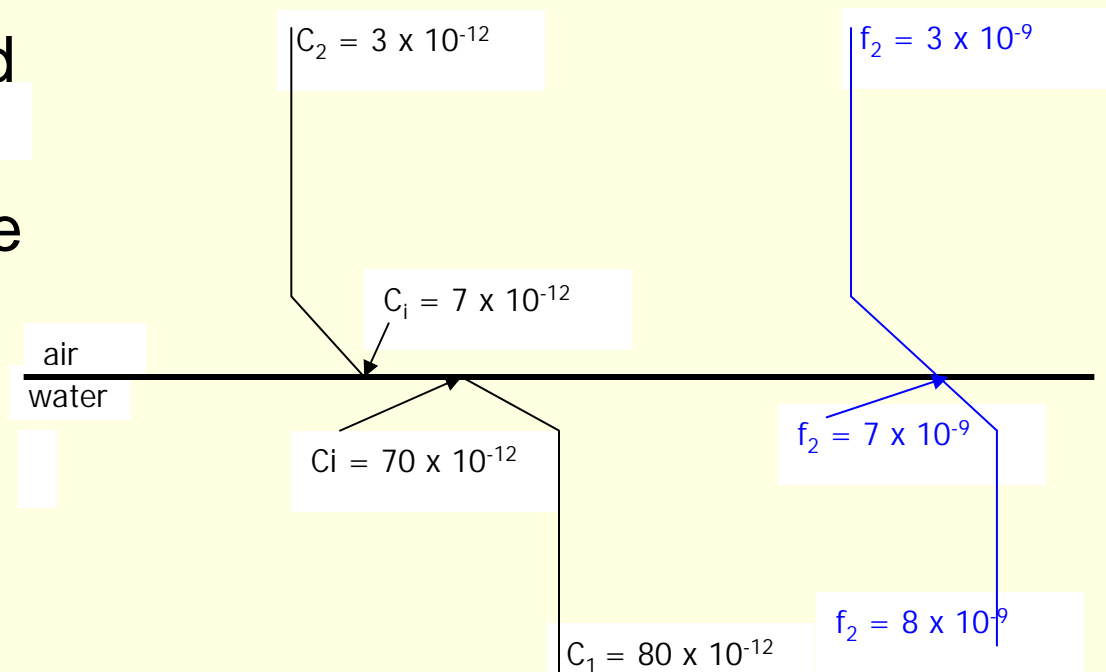
---

- Why Fugacity?
- Hydrodynamic model
- Contaminant Transport
- Solution of Governing Equations
  - Hydrodynamics
  - Contaminant Transport
- Application
- Future work

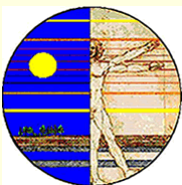


# Why Fugacity?

- Linear relationship between concentration and fugacity  $C = F Z$
- Continuous profile among different phases
- Decreases the number of coefficients used



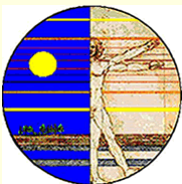
Source: Mackay, 1980



# Hydrodynamics of Rivers

---

- Characteristics of Channel Flow
  - Free surface
  - Gravity is the driving force
  - Always unsteady
- Development of unsteady flow equations for open channels
  - First published in 1871 by Saint Venant
  - Still used



# Derivation of Saint-Venant Equations for River Hydrodynamics

- Conservation of mass

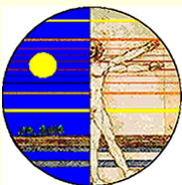
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

A – cross-sectional area ( $L^2$ )  
Q – volumetric flowrate ( $L^3/T$ )  
t – time (T)  
X – distance along the river (L)

- Conservation of momentum

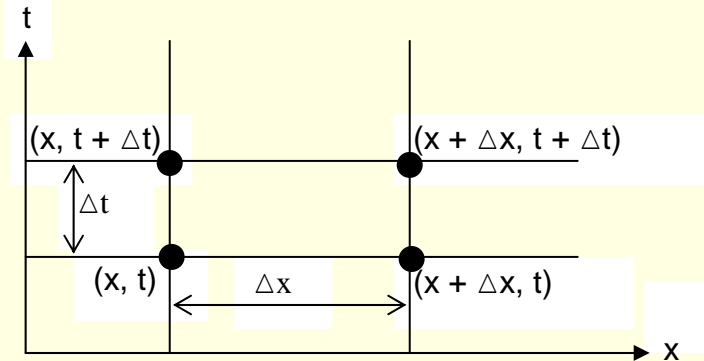
$$\frac{\partial Q}{\partial t} + \frac{\partial (Qu)}{\partial x} = -gA \frac{\partial h}{\partial x} + gA(S_o - S_f)$$

Q – flowrate,  $L^3/T$   
u – velocity,  $L/T$   
t – time, T  
A – area,  $L^2$   
x – distance along the river, L  
h – water surface elevation, L  
g – gravitational acceleration,  $L/T^2$   
 $S_o$  – bed slope, L/L  
 $S_f$  – friction slope L/L



# Solution of Saint-Venant Equations

- Coupled, non-linear, first order partial differential equations
- No analytical solution, so require numerical solution
- Numerical technique used in this study:
  - Preissmann Weighted Four-Point Method
    - Implicit scheme
    - Flexible time steps and distance
    - Unconditionally stable



$$\frac{\partial \phi}{\partial x} = \frac{(1-\theta)(\phi_{x+\Delta x}^t - \phi_x^t) + \theta(\phi_{x+\Delta x}^{t+\Delta t} - \phi_x^{t+\Delta t})}{\Delta x}$$

$$\frac{\partial \phi}{\partial t} = \frac{\left( \frac{\phi_x^{t+\Delta t} + \phi_{x+\Delta x}^{t+\Delta t}}{2} - \frac{\phi_x^t + \phi_{x+\Delta x}^t}{2} \right)}{\Delta t}$$

$$\phi = (1-\theta) \left( \frac{\phi_{x+\Delta x}^t + \phi_x^t}{2} \right) + \theta \left( \frac{\phi_{x+\Delta x}^{t+\Delta t} + \phi_x^{t+\Delta t}}{2} \right)$$



# Contaminant Transport:

- Advection Dispersion Equation is used

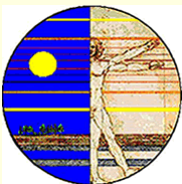
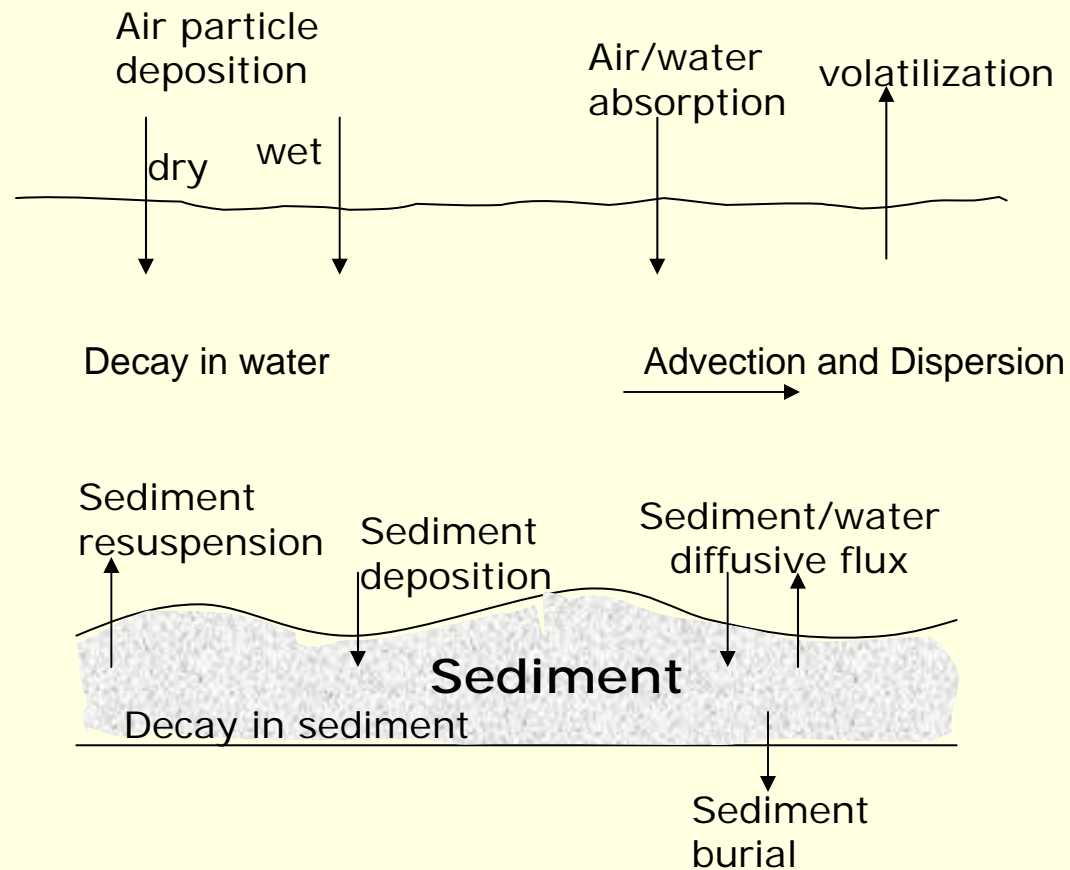
$$\frac{\partial C}{\partial t} + \frac{\partial(Cu_x)}{\partial x} - \frac{\partial}{\partial x} \left( D_{Hx} \frac{\partial C}{\partial x} \right) = \sum \text{Reaction}$$

- Reactions include:
  - Volatilization
  - Diffusion between air/water and water/sediment
  - Deposition/resuspension of suspended particles
  - Wet and dry deposition of aerosols from atmosphere
  - Decay of the chemical in water column and sediments





# Natural Processes in a River Reach



# Contaminant Transport

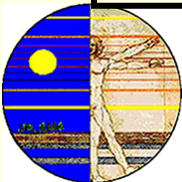
- Contaminants are distributed among all media in the vicinity of water column
- Sediments and air are also considered as a compartment in the contaminant transport model
- Sediments are immobile
- Air concentration is assumed to be constant

$$\frac{\partial(F_{water}Z_{bw})}{\partial t} + \frac{\partial(F_{water}Z_{bw}u_x)}{\partial x} - \frac{\partial}{\partial x} \left( D_{Hx} \frac{\partial F_{water}Z_{bw}}{\partial x} \right) = \sum \text{Reactions}$$

Water Phase

$$\frac{\partial(F_{sed}Z_{sw})}{\partial t} = \sum (\text{Reactions} + \text{interactions with water})$$

Sediment Phase



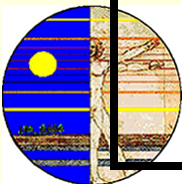
# Water Phase Fugacity Model:

$$\begin{aligned}
 V_w Z_{Bw} \frac{\partial f_w}{\partial t} = & V_w Z_{Bw} D_H \frac{\partial^2 f_w}{\partial x^2} - V_w Z_{Bw} U \frac{\partial f_w}{\partial x} + f_s \left( M \left( \frac{\tau_b}{\tau_c} - 1 \right) \frac{K_p \rho_s}{H} + \frac{A_w}{H \left( \frac{1}{K_w} + \frac{1}{K_s K_p \rho_s} \right)} \right) \\
 & + f_a \left( \frac{HA_w}{K_{aw}} + u_D \frac{A_w}{RT} + u_R \frac{QA_w}{RT} \right) - f_w \left( \frac{HA_w}{K_{aw}} + V_w \frac{k_w}{H} + \frac{gd^2}{18} \left( \frac{\rho_s - \rho}{\mu} \right) \frac{K_p \rho_s}{H} + \frac{A_w}{H \left( \frac{1}{K_w} + \frac{1}{K_s K_p \rho_s} \right)} \right)
 \end{aligned}$$

Dispersion
Advection
Resuspension
Diffusion

Diffusion
Dry and wet deposition
Volatilization
Decay
Settling
Diffusion

$$\frac{\partial f_w}{\partial t} = \frac{\partial}{\partial x} \left( D_H \frac{\partial f_w}{\partial x} \right) - \frac{\partial (u f_w)}{\partial x} + S_3 f_s + S_4 f_a - f_w S_5$$



# Sediment Phase Fugacity Model:

$$\begin{aligned}
 V_s Z_{Bs} \frac{\partial f_s}{\partial t} = f_w & \left( \underbrace{\frac{gd^2}{18} \left( \frac{\rho_s - \rho}{\mu} \right) \frac{K_p \rho_s}{H}}_{\text{Settling}} + \underbrace{\frac{A_w}{H \left( \frac{1}{K_w} + \frac{1}{K_s K_p \rho_s} \right)}}_{\text{Diffusion}} \right) \\
 - f_s & \left( \underbrace{M \left( \frac{\tau_b}{\tau_c} - 1 \right) \frac{K_p \rho_s}{H}}_{\text{Resuspension}} + \underbrace{\frac{A_w}{H \left( \frac{1}{K_w} + \frac{1}{K_s K_p \rho_s} \right)}}_{\text{Diffusion}} + \underbrace{V_{burial} \frac{K_p \rho_s}{H}}_{\text{Burial}} + \underbrace{V_s \frac{k_s K_p \rho_s}{H}}_{\text{Decay}} \right)
 \end{aligned}$$

$$\frac{\partial f_s}{\partial t} = S_6 f_w - S_7 f_s$$



# Contaminant Transport Model:

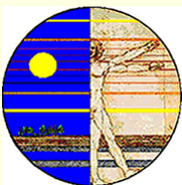
- The final aqueous fugacity balance:

$$\frac{\partial f_w}{\partial t} = \frac{\partial}{\partial x} \left( D_H \frac{\partial f_w}{\partial x} \right) - \frac{\partial(Uf_w)}{\partial x} + S_3 f_s + S_4 f_a - f_w S_5$$

$$\frac{\partial f_w}{\partial t} = D_H \frac{\partial^2 f_w}{\partial x^2} + \frac{\partial D_H}{\partial x} \frac{\partial f_w}{\partial x} - U \frac{\partial f_w}{\partial x} - f_w \frac{\partial U}{\partial x} + S_3 f_s + S_4 f_a - f_w S_5$$

- The final sediment fugacity balance:

$$\frac{\partial f_s}{\partial t} = S_6 f_w - S_7 f_s$$



# Numerical Solution:

- Only water phase is mobile
- Sediment phase is immobile
- Air is assumed to be semi infinite (constant air concentration)
- Velocity input comes from hydrodynamics
- Dispersion coefficient calculated using velocity

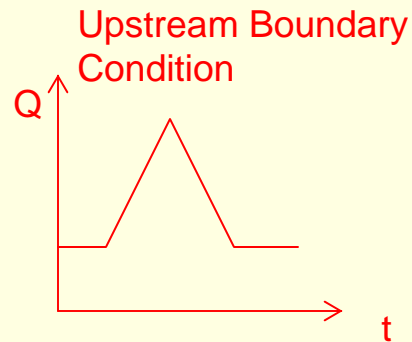
$$\frac{df_w}{dt} = \frac{f_{w_i}^{k+1} - f_{w_i}^k}{\Delta t}$$

$$\frac{df_w}{dx} = \frac{f_{w_{i+1}}^k - f_{w_{i-1}}^k}{2\Delta x}$$

$$\frac{d^2 f_w}{dx^2} = \frac{1}{2} \left[ \frac{f_{w_{i+1}}^{k+1} - 2f_{w_i}^{k+1} + f_{w_{i-1}}^{k+1}}{(\Delta x)^2} + \frac{f_{w_{i+1}}^k - 2f_{w_i}^k + f_{w_{i-1}}^k}{(\Delta x)^2} \right]$$



# Application:



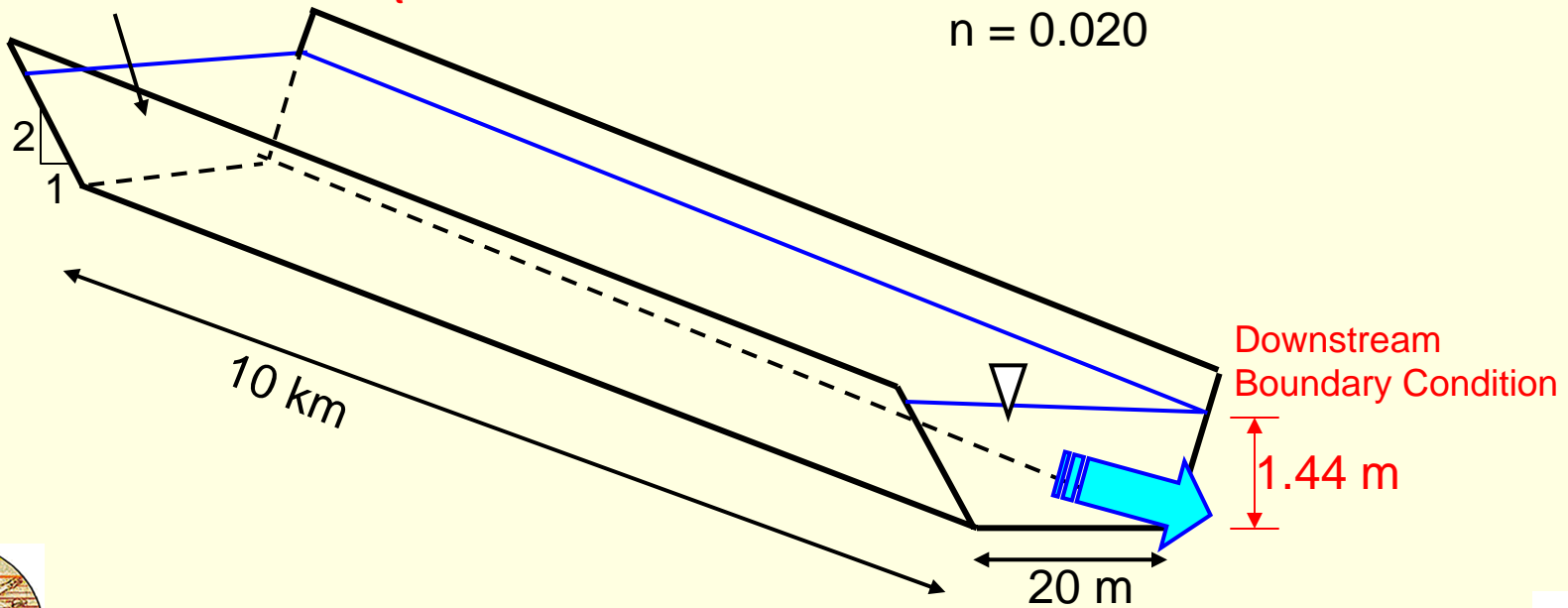
Total simulation time = 1 week

Number of nodes = 101

$\Delta x = 100$  m

$S_0 = 0.001$  m/m

$n = 0.020$



# Application:

---

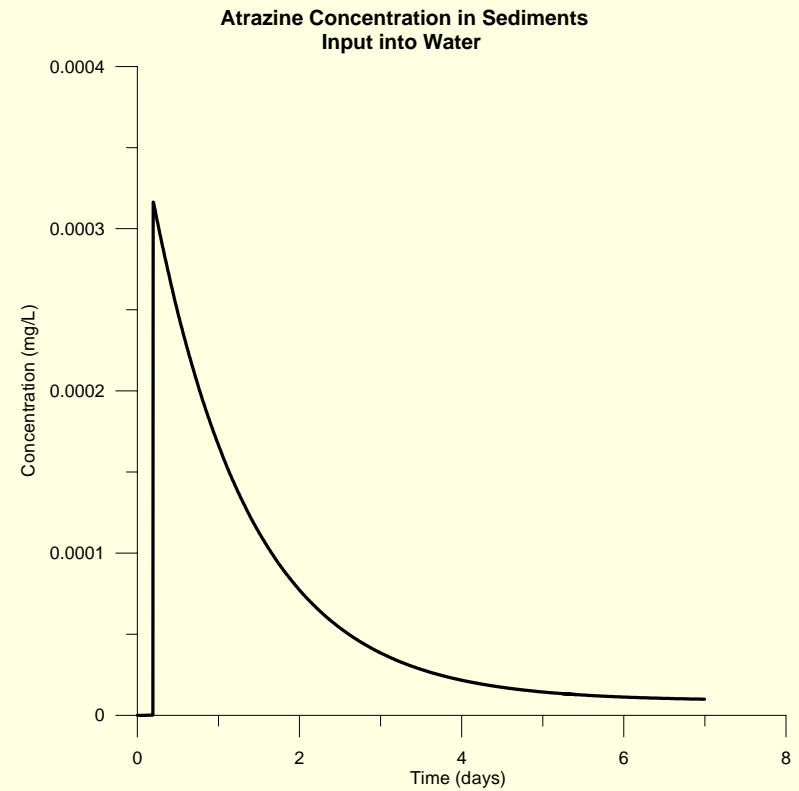
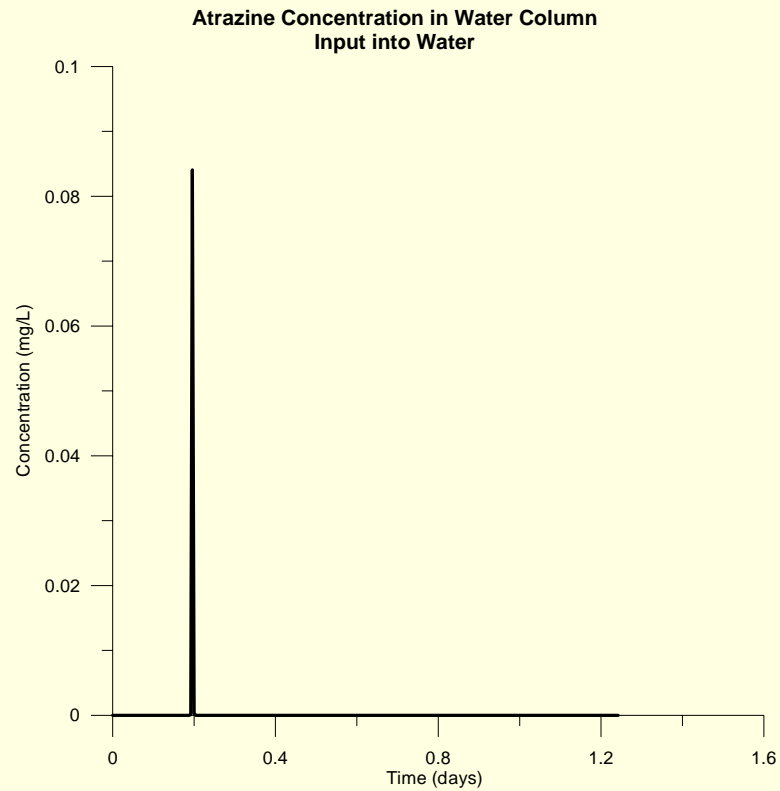
- Two application cases:
  - Case 1 : Instantaneous spill of atrazine into water column ( $C_0 = 0.1 \text{ mg/L}$ )
    - Atrazine is the most commonly used agricultural herbicide
  - Case 2: Sediment release of PCB as a result of dredging ( $C_0 = 0.1 \text{ mg/L}$ )
    - Although abandoned, PCBs are still existent in large amounts in the sediments of rivers





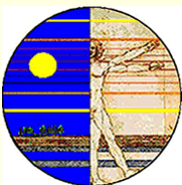
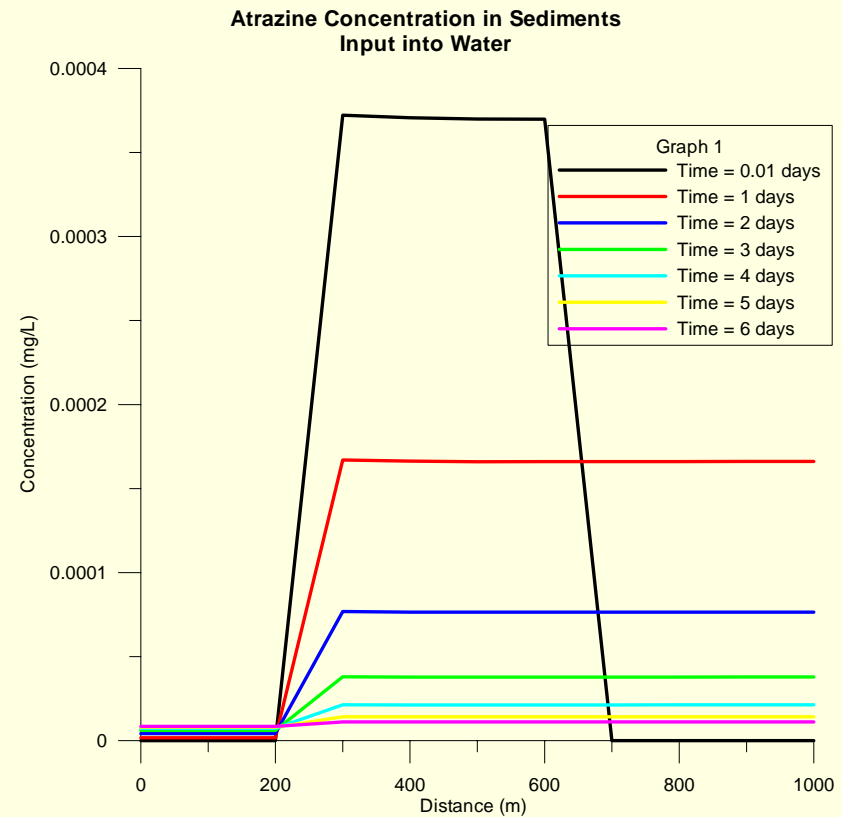
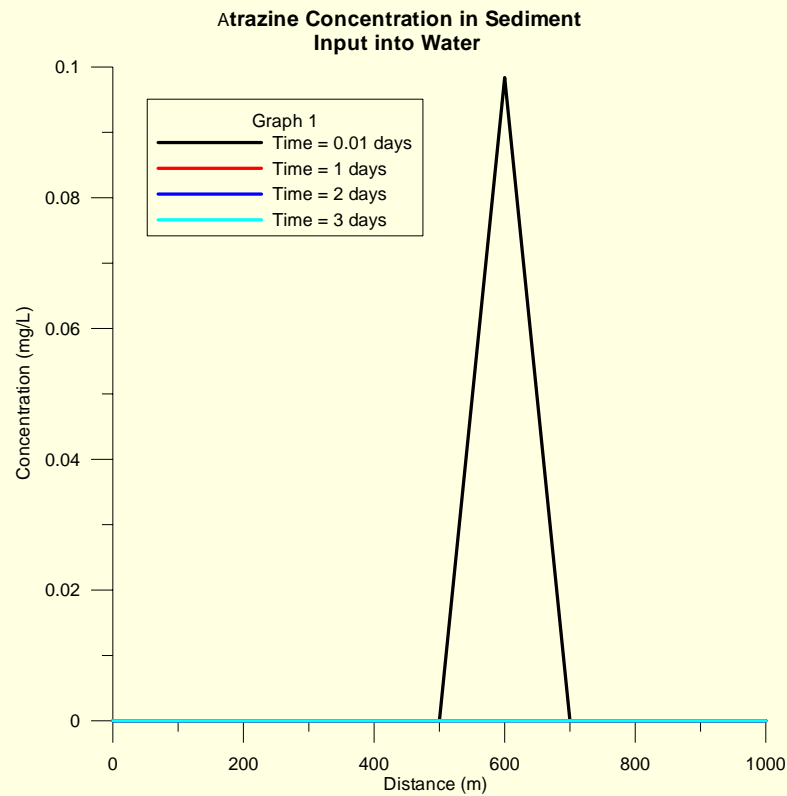
# Results – Case 1

## (Atrazine Spill into Water Column)



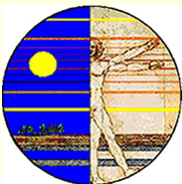
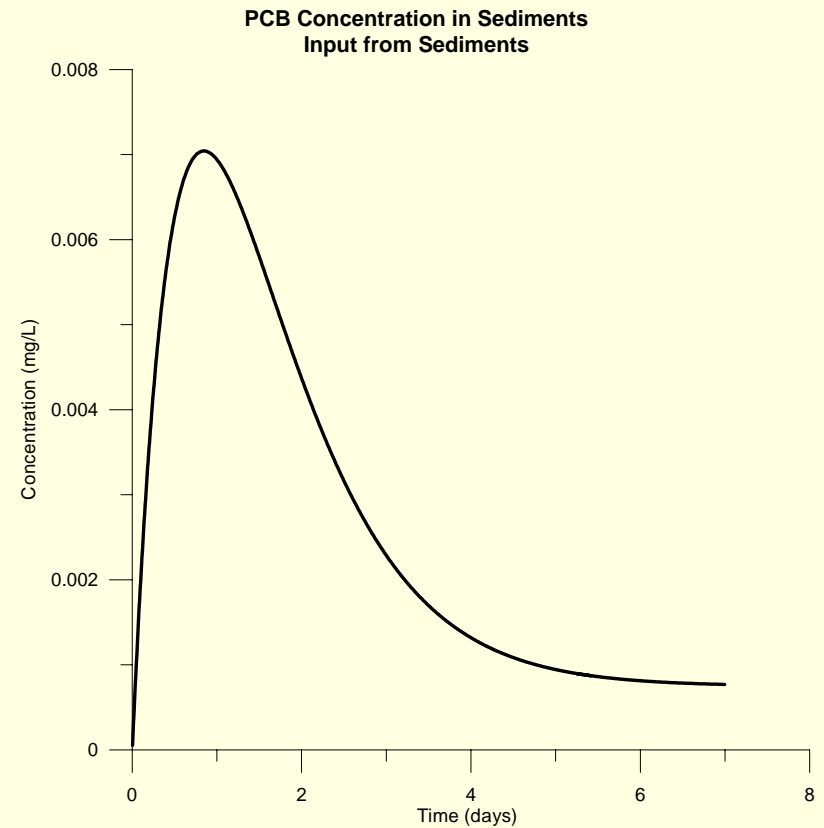
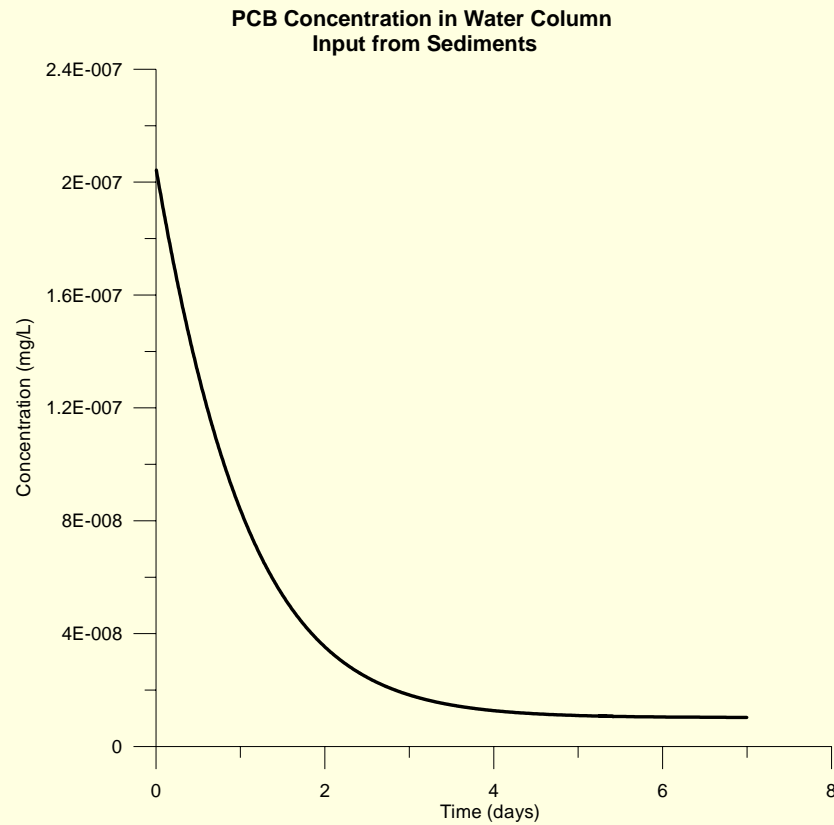
# Results – Case 1

## (Atrazine Spill into Water Column)



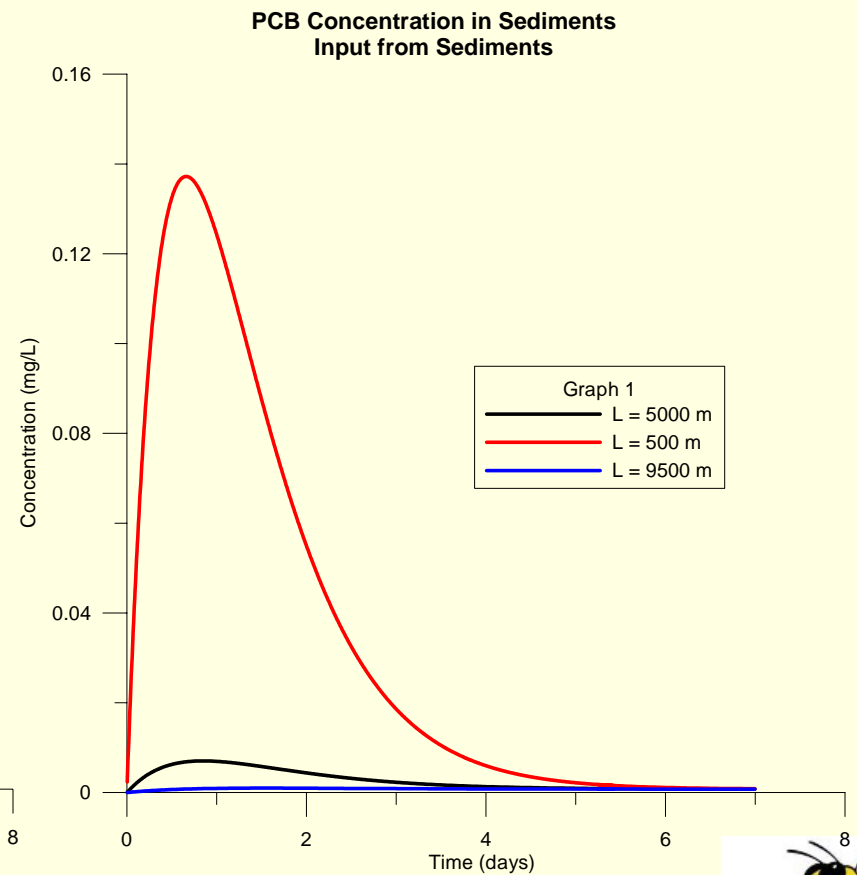
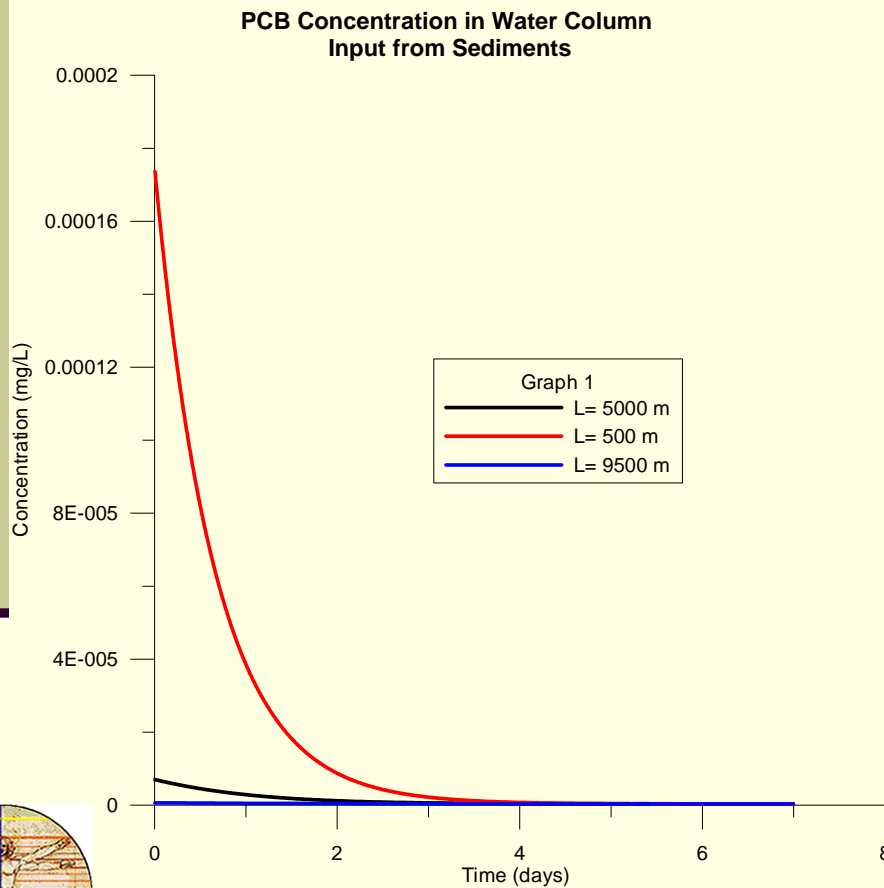
# Results – Case 2

## (PCB Release from Sediments)



# Results – Case 2

## (PCB Release from Sediments)



# Future Work

- A real case application
- Altamaha River

