An Integrated Model of Water Flow in Soil-Plant Systems

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Objectives

- Develop a model of water flow in a terrestrial system that
  - can describe water flow on the land surface and within the shallow soil for extended periods of time
  - takes into account the effects of the presence of vegetation on the overall water flow dynamics within the system
  - treats vegetation as a dynamic entity considering
    - its life cycle
    - its response to water availability within the system

- Develop a tool
  - for prediction of surface runoff
  - for prediction of infiltration
  - that can be further combined with contaminant transport models
Modeling Approach

- The modeling domain is represented by a collection of soil-plant system units.
- The soil-plant system units are coupled through
  - An *overland flow model* to represent ground surface flow processes.
  - Accounting for the *lateral flow fluxes* between the corresponding soil layers.
Schematic View of the Integration

Precipitation / Irrigation

Evaporation

Infiltration

Overland Flow

Soil columns

$z$ (depth)

$h_o$

$x$

$H$

$Z$
Schematic View of the Integration

- Precipitation / Irrigation
- Evaporation
- Infiltration
- Overland Flow

Variables:
- $h_0$
- $z$ (depth)
- $H$
Integrated Flow Model

- Overland flow model
  - 2D non-inertial-wave (diffusion wave) approximation to the Saint-Venant equations
- Unsaturated zone soil-water flow model coupled with the plant life-cycle model
- Subsurface lateral fluxes
  - Darcy fluxes between corresponding soil cells in adjacent columns
**Overland / Subsurface Interaction Algorithm**

\[
\begin{align*}
\left( R - E_{\text{surface}} \right) \Delta t + h_o & > I \Delta t ? \\
\left( R - E_{\text{soil}} \right) + 0, \frac{Q_L}{\Delta x \Delta y} + \frac{h_o}{\Delta t} & > 0 ? \\
\left( R - E_{\text{soil}} \right) + 0, \frac{Q_L}{\Delta x \Delta y} + \frac{h_o}{\Delta t} & > I ? \\
\left( R - E_{\text{soil}} \right) + 0, \frac{Q_L}{\Delta x \Delta y} + \frac{h_o}{\Delta t} & > E_{\text{max}} ?
\end{align*}
\]

**WET**

\[
\begin{align*}
h_{\text{top}} &= h_o \\
F &= ( R - E_{\text{surface}} - I ) \Delta x \Delta y
\end{align*}
\]

**DRY**

\[
\begin{align*}
h_{\text{top}} &= h_{\text{atm}} \\
F &= 0
\end{align*}
\]

**DRY - Transition**

\[
q_{\text{top}} = ( R - E_{\text{soil}} ) + 0, \frac{Q_L}{\Delta x \Delta y} + \frac{h_o}{\Delta t}
\]

\[
I = -K_{1/2} \left( \frac{h_1 - h_o}{0.5\Delta z_1} - 1 \right)
\]

**Note:**

- \( h_o \): soil-water pressure head in equilibrium with the prevailing relative humidity in the atmosphere [L]
- \( E_{\text{max}} \): maximum energy constraint
- \( F \): energy flux
- \( Q_L \): energy flux due to latent heat
- \( a \) and \( b \): parameters with \( \|a, b\| = a \)

\[
Q_L = (q_E + q_w) \Delta y + (q_S + q_N) \Delta x
\]

for \( a \geq b \), \( \|a, b\| = a \)
Subsurface Lateral Fluxes

Lateral flux into cell $j$ from EAST:

$$ q_j^E = K_j^E \frac{h_j^E - h_j}{\Delta x_{j,E}} $$

$K_j^E$ : The interblock hydraulic conductivity at the EAST interface of cell $j$. [L T$^{-1}$]
$h_j$ : Pressure head value at cell $j$. [L]
$h_j^E$ : Pressure head value at the EAST neighbor of cell $j$. [L]
$\Delta x_{j,E}$ : The distance between the centers of cell $j$ and its EAST neighbor. [L]

Spatially discretized soil-water flow equation with the lateral fluxes added:

$$ M_j \left( \frac{dh_j}{dt} \right)_j + S_j^1 h_{j-1} + S_j^2 h_j + S_j^3 h_{j+1} = F_j + q_j^E + q_j^W + q_j^N + q_j^S $$
LINEARIZATION OF THE vGM MODEL NEAR SATURATION

Richards’ Equation:

\[
C \frac{dh}{dt} = \frac{\partial}{\partial z} \left( K \left( \frac{\partial h}{\partial z} - 1 \right) \right) - U
\]

vGM Model:

\[
S_e(h) = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \left[ 1 + \left( \alpha_v |h| \right)^{n_v} \right]^{-n_v}
\]

\[
K(h) = K_s \left( S_e(h) \right)^{1/2} \left[ 1 - \left( 1 - \left( S_e(h) \right)^{1/m_v} \right)^{m_v} \right]^2
\]

\[
\theta_s = 0.41, \; \theta_r = 0.05, \; n_v = 1.31, \; \alpha_v = 1.9, \; K_s = 5.2 \times 10^{-7} \text{ m/s}
\]
LINEARIZATION OF THE vGM MODEL NEAR SATURATION

Richards’ Equation:

\[ \frac{dh}{dt} = \frac{\partial}{\partial z} \left( K \left( \frac{\partial h}{\partial z} - 1 \right) \right) - U \]

vGM Model:

\[ S_e(h) = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \left[ 1 + \left( \alpha_v |h| \right)^{n_v} \right]^{-n_v} \]

\[ K(h) = K_s \left( S_e(h) \right)^{1/2} \left[ 1 - \left( 1 - S_e(h) \right)^{1/m_v} \right]^{m_v} \]

Linearized \( K \): \( K(h)^* \)

\[ h - h^{0.99} = \frac{K(h)^* - K(h^{0.99})}{K_s - K(h^{0.99})} \]

\[ \theta_s = 0.41, \theta_r = 0.05, n_v = 1.31, \alpha_v = 1.9, K_s = 5.2 \times 10^{-7} \text{ m/s} \]
Effect of Vegetation on Overland Flow

The Manning’s roughness coefficient variation due to plant growth:

\[ n = (n_{\text{max}} - n_{\text{min}}) \left[ 1 - \exp(-LAI) \right] + n_{\text{min}} \]

adopted from Mailhol and Merot (2008)

- \( n \): Manning’s roughness coefficient. [T L^{-1/3}]
- \( n_{\text{max}} \): Maximum value of \( n \) when the crops are mature (when \( LAI = LAI_{\text{max}} \)). [T L^{-1/3}]
- \( n_{\text{min}} \): Minimum value of \( n \) when there are no crops (when \( LAI = 0 \)). [T L^{-1/3}]
- \( LAI \): Leaf area index value. [L^2 L^{-2}]

Graph showing the variation of \( n \) with \( LAI \).
Effect of Vegetation on System Hydrology

\[ E_p = (1 - f_c) E_{p,0} \]
\[ T_p = f_c E T_p - E_{\text{int}} \]
\[ S_{\text{int,max}} = \text{LAI} \cdot S_L \]

\( f_c \) : Vegetation cover fraction [L^2 L^{-2}]
\( E_{p,0} \) : Potential evaporation rate for bare, wet soil according to the site conditions [L T^{-1}]
\( E T_p \) : Potential evapotranspiration rate representing the combined effect of the evaporation and transpiration processes occurring at the site [L T^{-1}]
\( E_{\text{int}} \) : Evaporation rate from interception [L T^{-1}]
\( S_L \) : Specific storage capacity [L]
Model Application

The overland flow domain

<table>
<thead>
<tr>
<th>Slope in x-direction</th>
<th>0.0005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope in y-direction</td>
<td>0.0</td>
</tr>
<tr>
<td>Length in x-direction</td>
<td>400 m</td>
</tr>
<tr>
<td>Length in y-direction</td>
<td>400 m</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>Zero-depth gradient outlet at x = 400 m Other sides are no-flow boundaries</td>
</tr>
<tr>
<td>Initial condition</td>
<td>Dry</td>
</tr>
<tr>
<td>Spatial discretization, Δy</td>
<td>100 m (ny = 4)</td>
</tr>
<tr>
<td>Spatial discretization, Δx</td>
<td>100 m (nx = 4)</td>
</tr>
</tbody>
</table>
Model Application

Soil characteristics (silt loam):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated hydraulic conductivity, $K_{sat}$</td>
<td>$1.25 \times 10^{-6}$ m/s</td>
</tr>
<tr>
<td>van Genuchten parameters</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>1.41</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.0 l/m</td>
</tr>
<tr>
<td>$\theta_{res}$</td>
<td>0.067</td>
</tr>
<tr>
<td>$\theta_{sat}$</td>
<td>0.45</td>
</tr>
</tbody>
</table>

B.C.: free drainage

Soil Cell Thicknesses

Initial Condition:

Soil-water pressure head, $h$ (m)
### Plant Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corn</th>
<th>Hay</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Mailhol, O. et al., 1997; Wohling and Mailhol, 2007)</td>
<td>(Mailhol &amp; Merot, 2008)</td>
<td></td>
</tr>
<tr>
<td><strong>LAI related</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$LAI_{\text{max}}\ (m^2m^{-2})$</td>
<td>4.5</td>
<td>8</td>
</tr>
<tr>
<td>$T_b\ (\degree C)$</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>$T_s\ (\degree C)$</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>$T_f\ (\degree C)$</td>
<td>1005</td>
<td>900</td>
</tr>
<tr>
<td>$l$</td>
<td>1.25</td>
<td>10</td>
</tr>
<tr>
<td>$b$</td>
<td>2.4</td>
<td>3</td>
</tr>
<tr>
<td>$d_1$</td>
<td>14</td>
<td>3.7</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.2</td>
<td>4.4</td>
</tr>
<tr>
<td><strong>Biomass related</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{\text{mat}}\ (\degree C)$</td>
<td>1925</td>
<td>1020</td>
</tr>
<tr>
<td>$RUE\ (gMJ^{-1})$</td>
<td>1.32</td>
<td>0.37</td>
</tr>
<tr>
<td><strong>Root related</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_R(0)$</td>
<td>0.1</td>
<td>0.1 (assumed)</td>
</tr>
<tr>
<td>$L_{R,\text{max}}\ (m)$</td>
<td>1.2</td>
<td>0.5 (assumed)</td>
</tr>
<tr>
<td>$t_R\ (\text{days})$</td>
<td>72</td>
<td>50</td>
</tr>
<tr>
<td><strong>Evapotranspiration related</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{c,\text{max}}$</td>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>$x_k$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$k$</td>
<td>0.7</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Overland flow related</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_{\text{max}}$</td>
<td>0.15 (Assumed)</td>
<td>0.3</td>
</tr>
</tbody>
</table>
### Weather Data

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air temperature ($T$)</td>
<td>20°C</td>
</tr>
<tr>
<td>Reference evapotranspiration ($ET_0$)</td>
<td>2.5 mm/day</td>
</tr>
<tr>
<td>Reference evaporation rate from bare soil, ($E_{p,0}$)</td>
<td>2.0 mm/day</td>
</tr>
<tr>
<td>Evaporation rate from free water surfaces ($E_{w,0}$)</td>
<td>3.0 mm/day</td>
</tr>
<tr>
<td>$h_{atm}$</td>
<td>-160 m</td>
</tr>
</tbody>
</table>
Irrigation Schedule

- 6-hour irrigation period every 10 days starting at day 1.
- Two irrigation methods:
  - **Border irrigation**
    The water is input to the upper cells that are bordering the $x=0 \ m$ boundary.
  - **Sprinkler irrigation**
    The water is distributed uniformly to the *whole overland domain*.

<table>
<thead>
<tr>
<th></th>
<th>Border Irrigation</th>
<th>Sprinkler irrigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge rate (mm/hour)</td>
<td>13.5</td>
<td>3.375</td>
</tr>
<tr>
<td>Discharge time (hours)</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Border cells area (m²)</td>
<td>40000</td>
<td>160000</td>
</tr>
<tr>
<td>Discharge Volume (m³)</td>
<td>3240</td>
<td>3240</td>
</tr>
</tbody>
</table>
Classification of the Simulations

- **Simulation Length**
  - Daily (simulation length = 1 day)
  - Seasonal (simulation length = 120 days)

- **Irrigation Method**
  - Border Irrigation
  - Sprinkler Irrigation

- **Plant Simulation**
  - No Plant (Bare soil)
  - Const. Plant (corn: $LAI = 3.0$, hay: $LAI = 5.0$)
  - Plant (Plant life-cycle simulated)
Daily Simulations
BORDER IRRIGATION (No Plant)

Overland Flow Depth Variation with Time

Pressure Head Variation at the Top Soil Cell

Soil Cell 1
Daily Simulations
BORDER IRRIGATION (Const. Plant)

Overland Flow Depth Variation with Time

Pressure Head Variation at the Top Soil Cell
BORDER
Irrigation
(No Plant)
BORDER Irrigation (Plant)

Column 0, 1: Corn, LAI = 3.0

Column 2, 3: Hay, LAI = 5.0
Daily Simulations
SPRINKLER IRRIGATION

NO PLANT

Soil Cell 1

Pressure head, \( h \) (m)

Time (hours)

Column 0, 1: Corn, \( LAI = 3.0 \)
Column 2, 3: Hay, \( LAI = 5.0 \)

Soil Cell 1

Pressure head, \( h \) (m)

Time (hours)
Daily Simulation
SPRINKLER
Irrigation
(No Plant)
Daily Simulation
SPRINKLER Irrigation (Plant)
Seasonal Simulations

BORDER Irrigation (No Plant)
Seasonal Simulations
BORDER Irrigation (Const. Plant)
Seasonal Simulations
BORDER Irrigation (Plant)
Border Irrigation (No Plant)

Col. 0

Col. 1

Col. 2

Col. 3
Border Irrigation (Const. Plant)
Border Irrigation (Plant)
BORDER Irrigation

Col. 0

Col. 1

Col. 2

Col. 3
Seasonal Simulations
Sprinkler Irrigation (No Plant)

No Plant

Const. Plant

Plant
Sprinkler Irrigation (No Plant)
Sprinkler Irrigation (Const. Plant)
Sprinkler Irrigation (Plant)
SPRINKLER Irrigation

Col. 0

Col. 1

Col. 2

Col. 3
2× Enhanced Irrigation BORDER (No Plant)
2× Enhanced Irrigation BORDER (Const. Plant)
2× Enhanced Irrigation BORDER (Plant)
2× Border Irrigation (No Plant)
2× Border Irrigation (Const. Plant)
2× Border Irrigation (Plant)
2x BORDER Irrigation

Col. 0

Col. 1

Col. 2

Col. 3
2× Enhanced Irrigation SPRINKLER

No Plant

Const. Plant

Plant
2× Sprinkler Irrigation (No Plant)
2× Sprinkler Irrigation (Const. Plant)
2× Sprinkler Irrigation (Plant)
2x
SPRINKLER Irrigation

Col. 0

Col. 1

Col. 2

Col. 3
Conclusions

- A model was developed that
  - can describe water flow dynamics on the land surface and within the shallow soil for extended periods of time
  - takes into account the effects of the presence of vegetation on the overall water flow dynamics within the system

- The developed model is
  - able to provide an integrated analysis of water flow dynamics in terrestrial systems.
  - applicable to fields that have heterogeneous vegetation and soil characteristics.

- Model limitations:
  - The model is applicable to fields with relatively mild topography.
  - Groundwater and surface waters are out of the modeling domain.
Thank you...