Optimal Sensor Placement for Wind-Driven Lakes

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Needs to design a water quality monitoring system

- Requirement of water quality monitoring increases due to increasing health risk from degrading water quality.
  - Setting up and running a monitoring network is very costly and the performance depends on its design.
    - Monitoring locations are one of the most important factors.
  - However, a clear strategy to design a water quality monitoring network does not exist.
Where are the best monitoring locations? And How many?
Optimization of Sensor Locations

- Objective
  - Minimize detection time
    - Early detection allows more time to respond and helps to maximize usability of water body.
  - Decision variables
    - Sensor locations
  - Objective function
    \[
    \min f(X) = \frac{1}{S_n} \sum_{i=1}^{S_n} t_d^{s_i} (X, t_{r_i}^{s_i}, x_{r_i}^{s_i})
    \]

\( t_d^{s_i} (X, t_{r_i}^{s_i}, x_{r_i}^{s_i}) \): Detection time of scenario \( s_i \)
\( t_{r_i}^{s_i} \): Time of contaminant release of scenario \( s_i \)
\( x_{r_i}^{s_i} \): Location of contaminant release of scenario \( s_i \)
\( x_i \): Location of a sensor
\( X = \{x_1, \ldots, x_r, \ldots, x_n\} \): Sensor distribution
\( S_n \): Number of all scenarios
Procedure

- Set up scenarios
- Simulate hydrodynamics
  - Forcing for hydrodynamics
  - Source locations, release times
- Simulate contaminant transport scenario using the finite element method
- Calculate detection time at possible sensor locations
- Optimize sensor locations to minimize detection time using the genetic algorithm
Simulation of Physics

- Shallow water simulation
  \[ \frac{\partial h}{\partial t} + \frac{\partial (hU)}{\partial x} + \frac{\partial (hV)}{\partial y} = 0 \]
  \[ \frac{\partial (hU)}{\partial t} + \frac{\partial (hUU)}{\partial x} + \frac{\partial (hUU)}{\partial y} - E_h \left( \frac{\partial^2 (hU)}{\partial x^2} + \frac{\partial^2 (hU)}{\partial y^2} \right) + gh \frac{\partial \eta}{\partial x} - \tau_{xx} + \tau_{wx} = 0 \]
  \[ \frac{\partial (hV)}{\partial t} + \frac{\partial (hUV)}{\partial x} + \frac{\partial (hVV)}{\partial y} - E_h \left( \frac{\partial^2 (hV)}{\partial x^2} + \frac{\partial^2 (hV)}{\partial y^2} \right) + gh \frac{\partial \eta}{\partial y} - \tau_{yy} + \tau_{wy} = 0 \]

- Contaminant transport simulation
  \[ \frac{\partial (hC)}{\partial t} + \frac{\partial (UhC)}{\partial x} + \frac{\partial (VhC)}{\partial y} = \frac{\partial}{\partial x} \left( D_{xx} \frac{\partial (hC)}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_{yy} \frac{\partial (hC)}{\partial y} \right) + \frac{\partial}{\partial x} \left( D_{yx} \frac{\partial (hC)}{\partial y} \right) + \frac{\partial}{\partial y} \left( D_{xy} \frac{\partial (hC)}{\partial x} \right) \]

- Finite Element Method on a parallel computing environment

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Optimization – Genetic Algorithm

- Genetic algorithm
  - Suitable for complex objective functions
    - Does not require any gradient calculation
    - Does not fall into local optima easily
- Genetic algorithm can be parallelized
  - Island model
    - Helps to keep diversity
- The optimization routine searches the best nodes instead of $x$ and $y$-coordinate.
Chromosome and Evaluation

- Each gene represents a nodal index in a mesh, where a sensor is placed.
  - Simpler and more straightforward than a bit-string
    - The problem becomes an integer programming.
- Evaluation of fitness: Calculate how long it takes for a contaminant to reach to one of sensors.
Test Problem - Circular Lake

- To test the ability of the proposed approach, a circular lake with a wind-driven flow is set up and tested.
  - Deep at the center, shallow along the shoreline
  - Wind from the east
  - Flow reaches to a steady state.
  - Elements on the shoreline are regarded as possible source locations.
Circular Lake – Flow Pattern

- 0.1 m/s
  - Flow velocity

- Wind 10 m/s

X (m)  Y (m)
Circular Lake – Contaminant Transport

1 hr

4 hrs 40 min

8 hrs

12 hrs
Circular Lake – Optimized Locations

3 sensors

4 sensors
Lake Pontchartrain – Flow Pattern

- Similar setting as the circular lake case
Lake Pontchartrain – Optimized Locations

- Evenly distributed along the shoreline regardless of wind direction

Wind from the south

Wind from the east

Wind from the south and east
Lake Pontchartrain
– Average detection time vs. # of sensors

- As the number of sensor increases, the average detection time decreases.
- Provide trade-off between detection time and the number of sensors (= cost)
Conclusions

- Locations of monitoring sensors are critical factors of the performance of monitoring sensors.
- The Combination of numerical simulation and genetic algorithm shows promising results on a realistic way.
  - This approach may provide valuable information for decision makers to set up a monitoring system.
- In wind-driven lakes with a steady-state, best sensor locations will be places along a shoreline.
Current Work

- Finding a best path to measure, instead of several fixed sensor locations
  - Measurement using a remote-controlled boat may be an easier and dynamic solution.
- Optimization of Multiple objectives
  - # of sensors
  - What part of a domain should be protected more
- Unsteady state problems are being solved.