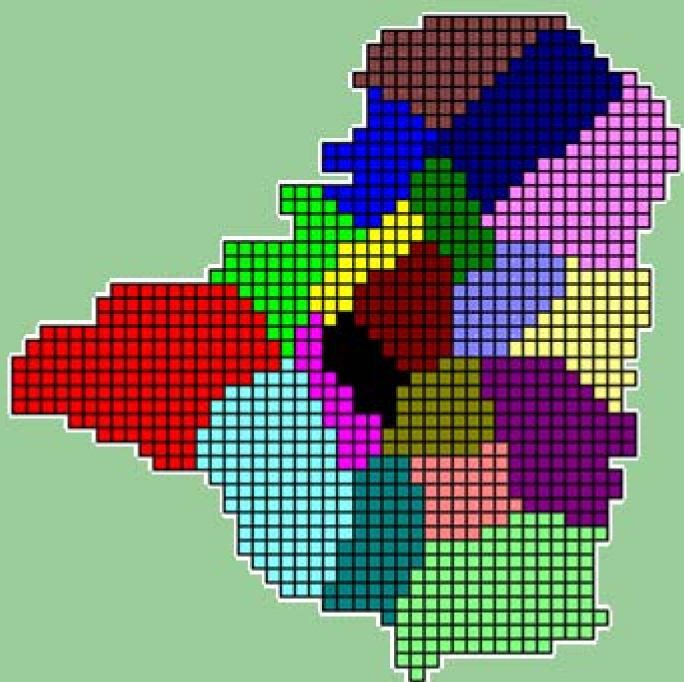
Simultaneous estimation of Aquifer Parameters and Parameter Zonations using Genetic Algorithm



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OUTLINE

- Objective
- Model Development
  - Parameter Estimation and Inverse Models
  - Simulation
  - Optimization with Genetic Algorithms
  - Pattern Classification
  - Problem Formulation and Search Procedure
- Numerical Example
- Conclusions

OUTLINE

Objective

Model Development

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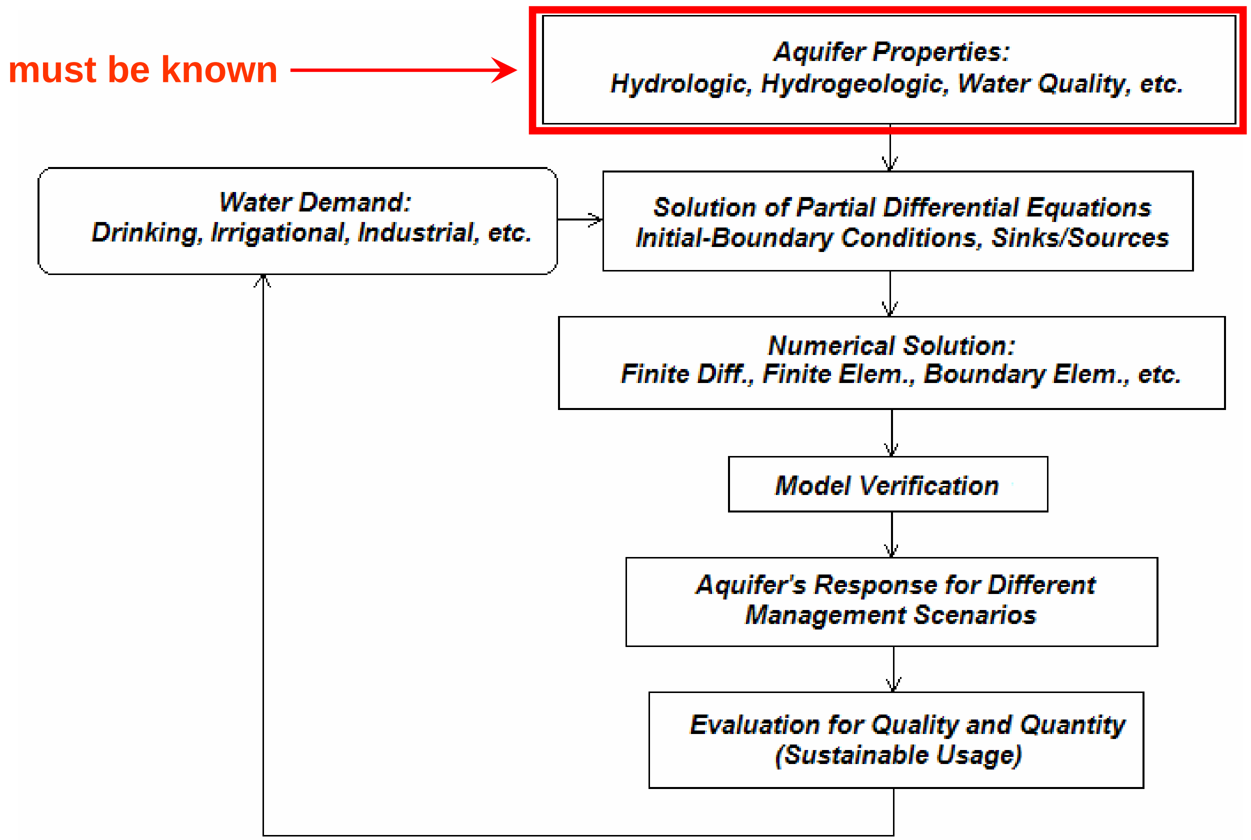
Numerical Example

Conclusions

Objective

- Mathematical simulation models are used to analyze and determine the sustainable usage and management of groundwater systems.
- These models require many hydrogeologic parameters including transmissivity, storativity, specific yield and recharge/infiltration.
- These aquifer parameters are generally obtained from laboratory experiments and/or field studies.
- Measurement and estimation of these spatially distributed parameters in large aquifer systems are time consuming and costly.
- In general, obtaining the hydraulic heads field data is relatively easier and cheaper than obtaining other parameters.
- Estimation of aquifer parameters based on limited field observations (for example piezometric head) is known as inverse modeling.

Objective


must be known
Aquifer Properties:
Hydrologic, Hydrogeologic, Water Quality, etc.
Water Demand:
Drinking, Irrigational, Industrial, etc.
Solution of Partial Differential Equations
Initial-Boundary Conditions, Sinks/Sources
Numerical Solution:
Finite Diff., Finite Elem., Boundary Elem., etc.
Model Verification
Aquifer's Response for Different
Management Scenarios
Evaluation for Quality and Quantity
(Sustainable Usage)

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Model Development

Parameter Estimation and Inverse Models

- In order to solve groundwater problems, two methods are widely used:

  Forward Model

  Inverse Model

- *The forward models are used to obtain hydraulic heads for given aquifer parameters (i.e. transmissivity, storativity, etc.)*
- *The inverse models are used to back out underlying model parameters using the hydraulic heads .*

Model Development

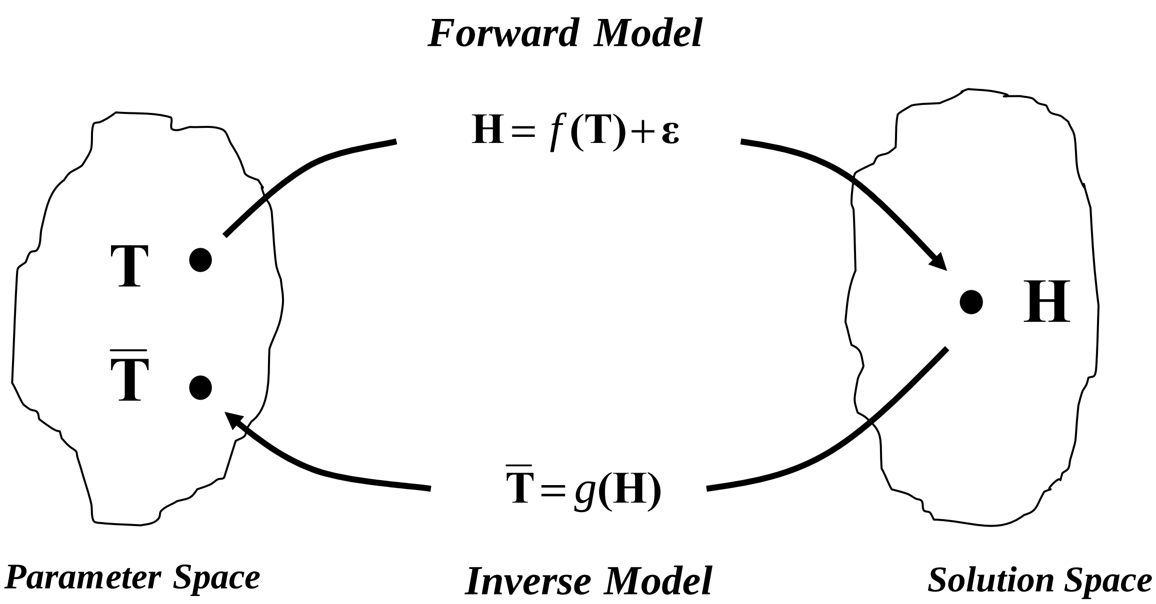
Parameter Estimation and Inverse Models

- There are several inherent difficulties with inverse solution techniques.
- One of the major difficulties in inverse modeling problems is their ill-posed nature.
- Ill-posed nature is usually characterized by the instability in inverse numerical solutions and non-uniqueness of the identified parameters.
- The instability and non-uniqueness of the inverse problems occur when small errors are introduced to the observed values (or numerical errors)
- This situtation causes large errors in the values of the parameters identified.

Model Development

Parameter Estimation and Inverse Models

- The relation between hydraulic heads and transmissivities for both forward and inverse models:



f : ***Forward Operator***

ε : ***Error Vector***

g_i : ***Generalized Inverse Operator***

$\varepsilon \rightarrow 0 \quad \Rightarrow \mathbf{T} = \overline{\mathbf{T}}$

Model Development

Parameter Estimation and Inverse Models

Several solution techniques have been used in order to solve inverse problems during the past two decades

The first group of solutions:

Statistical solution methods and Recursive filtering techniques

The second group of solutions:

Combination of several optimization approaches with numerical solution techniques

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Model Development

Simulation

- Simulation phase can defined as the numerical solution of the governing partial differential equation for a given initial and boundary conditions.
- The governing equation that describes flow in a two-dimensional, unconfined, compressible, non-homogeneous aquifer may be given as:

$$S\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)\pm W$$

Model Development

Simulation

- The following finite difference equation for heterogeneous material properties and non-uniform grid scheme is used for the solution:

$$h_{i,j}^{m+1,n+1} = \frac{\left[CC.(h_{i,j}^{m+1,n}) + CE.(h_{i+1,j}^{m+1,n}) + CW.(h_{i-1,j}^{m+1,n+1}) + CS.(h_{i,j+1}^{m+1,n}) + CN.(h_{i,j-1}^{m+1,n+1}) + W_{i,j}\right]}{(CE + CW + CS + CN + CC)}$$

$$CE = \frac{2T_{i,j}T_{i+1,j}}{(\Delta x_{(i)}T_{i+1,j} + \Delta x_{(i+1)}T_{i,j})\Delta x_{(i)}}$$

$$CC = \frac{S_{i,j}}{\Delta t}$$

$$CW = \frac{2T_{i,j}T_{i-1,j}}{(\Delta x_{(i-1)}T_{i,j} + \Delta x_{(i)}T_{i-1,j})\Delta x_{(i)}}$$

$$CN = \frac{2T_{i,j}T_{i,j-1}}{(\Delta y_{(j-1)}T_{i,j} + \Delta y_{(j)}T_{i,j-1})\Delta y_{(j)}}$$

$$W_{i,j} = \pm \frac{Q_{i,j}^{m+\frac{1}{2}}}{\Delta x_{(i)}\Delta y_{(j)}H_{i,j}}$$

$$CS = \frac{2T_{i,j}T_{i,j+1}}{(\Delta y_{(j)}T_{i,j+1} + \Delta y_{(j+1)}T_{i,j})\Delta y_{(j)}}$$

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Model Development

Optimization with Genetic Algorithms

Genetic Algorithms:

- efficient in continious and discrete optimization
- gives good results in finding global or near global optimum solutions
- does not require to evaluate the derivatives of the fitness function
- does not require the information on an initial solution
- finds the solution as a random search based on “survival of the fittest” concepts of the GA.

In GA, variables are usually represented by binary string chromosomes

Each chromosome is evaluated by using genetic operators (Direct Selection, Selection, Crossover and Mutation)

Model Development

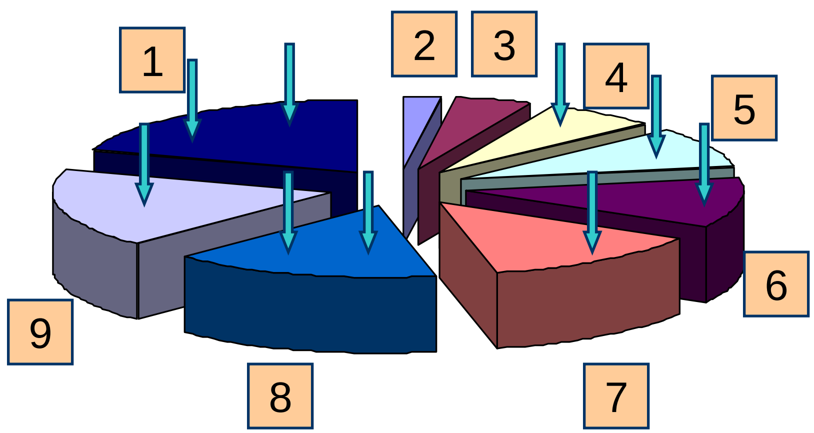
Optimization with Genetic Algorithms

Variable (Decimal)	X_1	X_2	X_3	...
Variable (Binary String)	1010…01	110…10	010…01	…

Model Development

Optimization with Genetic Algorithms

Selection (Roulette Wheel)



Before Selection

1	1	0	0	0	1	1	0	1	0	1	1
2	1	0	0	1	0	1	0	1	1	1	0
3	1	1	0	0	1	0	1	0	1	1	0
4	1	1	1	1	0	1	0	0	0	0	1
5	1	0	1	0	0	1	1	1	0	1	0
6	1	1	1	0	0	1	0	1	1	0	0
7	1	0	1	0	1	0	1	0	1	0	1
8	1	0	1	0	0	1	0	1	1	1	0
9	1	1	0	0	1	0	0	1	1	0	1

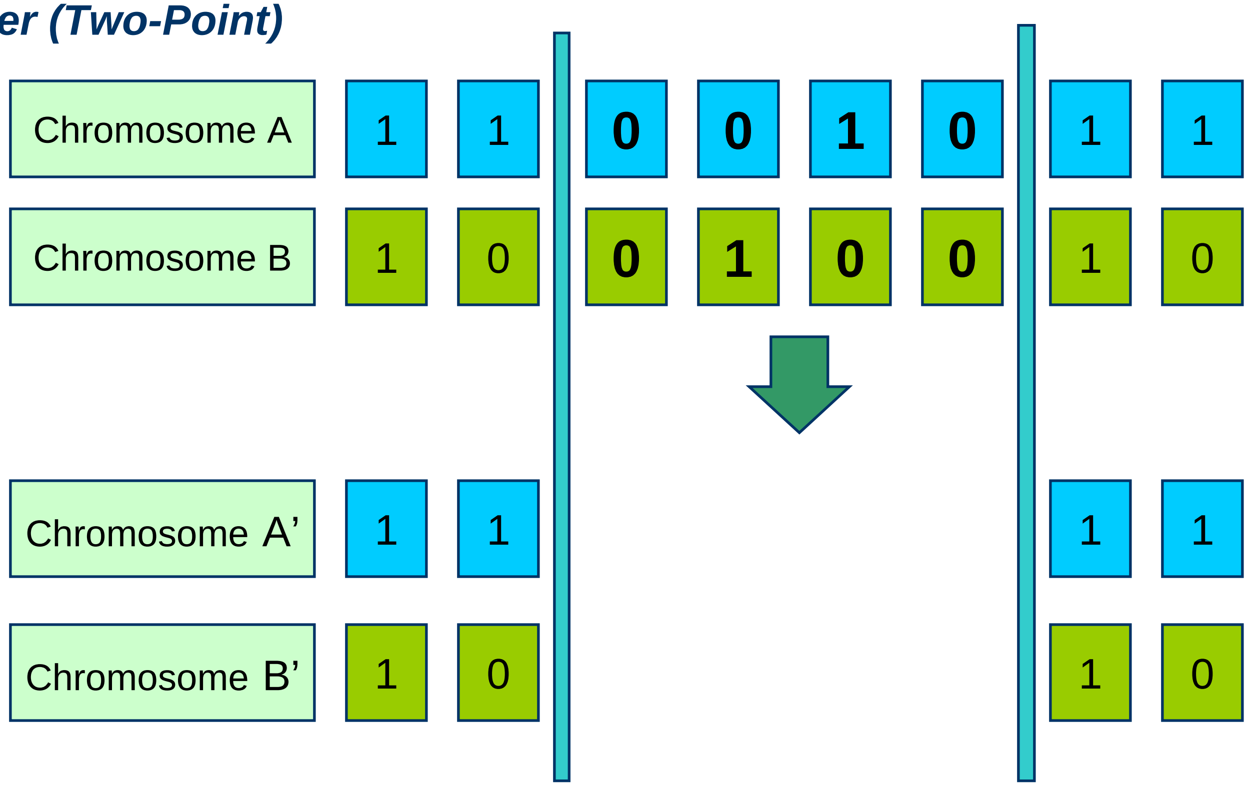
After Selection

1	1	0	0	0	1	1	0	1	0	1	1
1	1	0	0	0	1	1	0	1	0	1	1
4	1	1	1	1	0	1	0	0	0	0	1
5	1	0	1	0	0	1	1	1	0	1	0
6	1	1	1	0	0	1	0	1	1	0	0
7	1	0	1	0	1	0	1	0	1	0	1
8	1	0	1	0	0	1	0	1	1	1	0
8	1	0	1	0	0	1	0	1	1	1	0
9	1	1	0	0	1	0	0	1	1	0	1

Model Development

Optimization with Genetic Algorithms

Crossover (Two-Point)



Model Development

Optimization with Genetic Algorithms

Mutation (Adaptive)

Chromosome A’	1	1	**0**	**1**	**0**	**0**	1	1

Model Development

Optimization with Genetic Algorithms

Diversity Control of Population

While Number of Generation > Stall Generation Limit

If $\frac{f_j - f_{j-st}}{f_j} \le \varepsilon$ Then

Add initially generated chromosomes to the population by saving the elite values

End if

Wend

f_j : Fitness at the jth generation

f_{j-st} : Fitness at the (j - st)th generation

st : Stall generation limit

ε : Tolerance

Model Development

Optimization with Genetic Algorithms

Error Indicators

Modified Coefficient of Efficiency (E)

$$E = 1 - \frac{\sum_{l=1}^{L} \left| h_{obs,l} - h_{est,l} \right|}{\sum_{l=1}^{L} \left| h_{obs,l} - \overline{h_{obs}} \right|}$$

h_{obs} : Observed hydraulic heads

h_{est} : Estimated hydraulic heads

$\overline{h_{obs}}$: Mean of the observed hydraulic heads

L : Number of observation wells

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Model Development

Pattern Classification

• Patterns can be classified based on the values of membership functions.

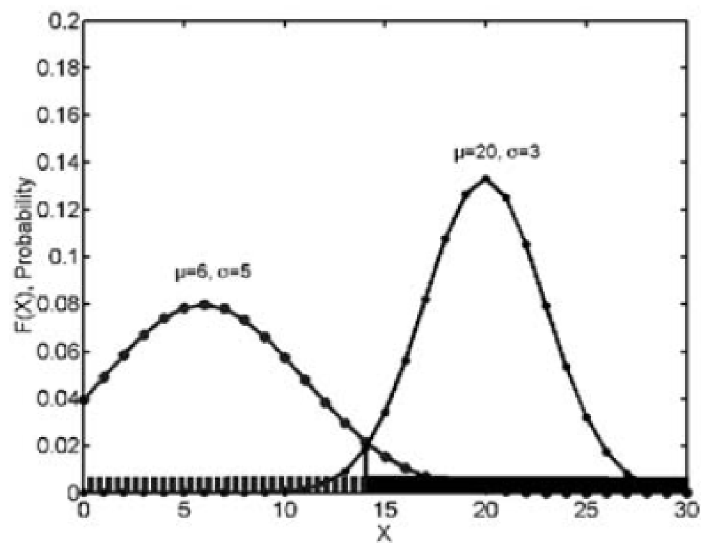
• For example, in a one-dimensional problem with two pattern types, two density functions of normal distribution can be used to separate the two groups.

$$N_i(X,\ \mu_i,\ \sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i}\exp\left[-\frac{1}{2}\left(\frac{X-\mu_i}{\sigma_i}\right)^2\right] \quad i = \mathrm{I}\ \&\ \mathrm{II}$$

μ_i : Mean and the location of the centroid of pattern i

σ_i : Standard deviation

X : Position of the grid point

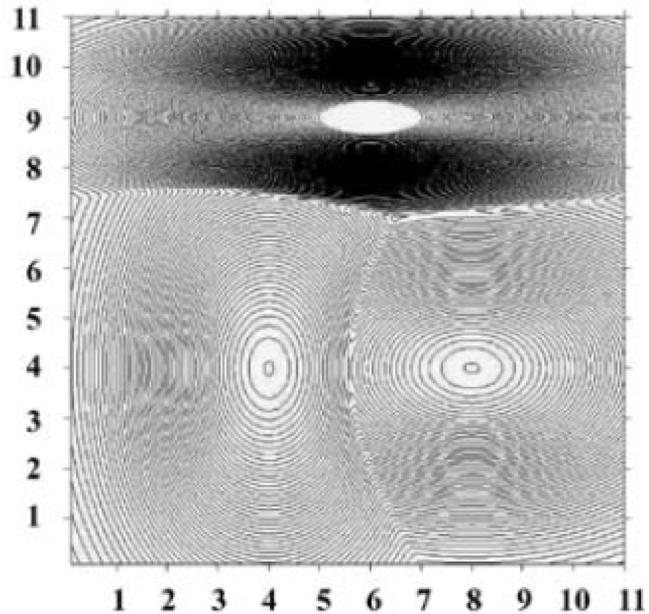


Model Development

Pattern Classification

Two-dimensional problem can use two dimensional density function of normal distribution to determine patterns as follows:

$$N_i(X, Y, \mu_{Xi}, \mu_{Yi}, \sigma_{Xi}, \sigma_{Yi}) = \frac{1}{2\pi\sigma_{Xi}\sigma_{Yi}}$$
$$\exp\left\{-\frac{1}{2}\left[\left(\frac{X-\mu_{Xi}}{\sigma_{Xi}}\right)^2 + \left(\frac{Y-\mu_{Yi}}{\sigma_{Yi}}\right)^2\right]\right\}$$



How can we determine the pattern???

Model Development

Pattern Classification

A grid point $P(X, Y)$ can be classified to pattern k, if the N_k yields the maximum value among n functions as seen in the following:

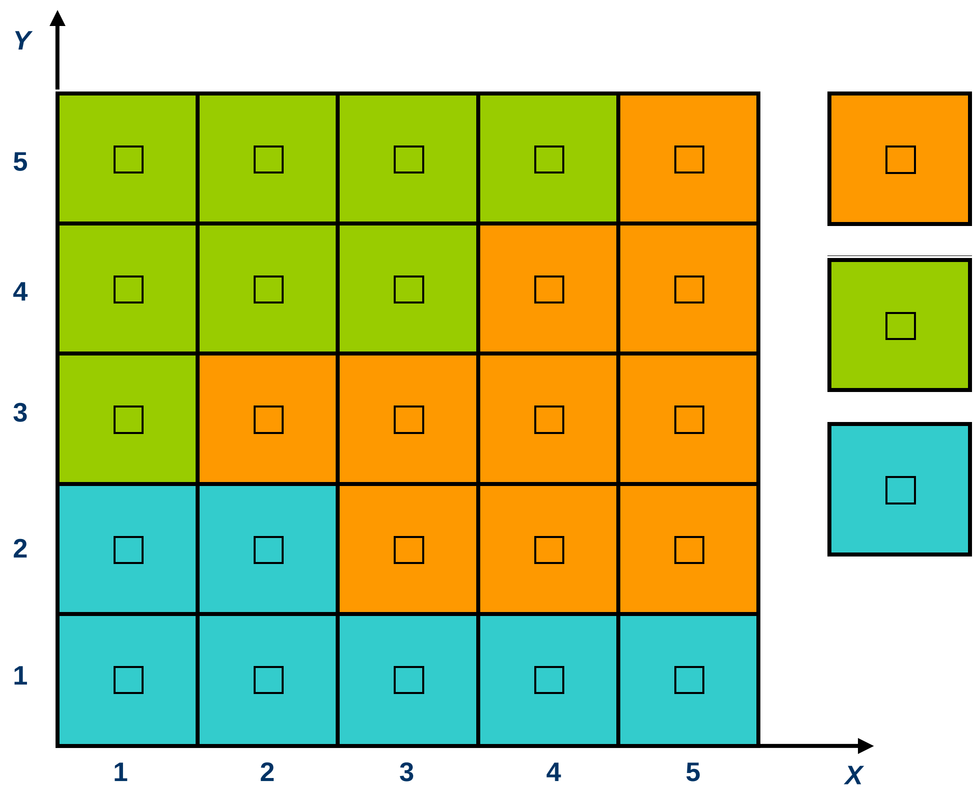
$$P(X, Y) \in \text{Pattern k}, \;\text{if}\; N_{\text{k}} = \text{Max}[N_{\text{I}},\; N_{\text{II}},\; \cdots N_{\text{k}},\; \cdots,\; N_{\text{n}}]$$

Variables of the each pattern to be optimized: $(\mu_{Xi},\; \mu_{Yi}, \sigma_{Xi}\;\; \sigma_{Yi})$

How does it work???

Model Development

Pattern Classification



$$N_I = f\left(\mu_{X1},\ \mu_{Y1},\ \sigma_{X1},\ \sigma_{Y1}\right)$$

$$\boxed{N_{II} = f\left(\mu_{X2},\ \mu_{Y2},\ \sigma_{X2},\ \sigma_{Y2}\right)}$$ Max

$$N_{III} = f\left(\mu_{X3},\ \mu_{Y3},\ \sigma_{X3},\ \sigma_{Y3}\right)$$

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Model Development

Problem Formulation and Search Procedure

The mathematical model can be summarized as follows:

$$Min\ Z = \frac{1}{L}\sum_{t=1}^{N_t}\sum_{l}^{L}\left|h_{obs,l}^{t} - h_{est,l}^{t}\right|$$

subject to

$$S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(\bar{T}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\bar{T}\frac{\partial h}{\partial y}\right) \pm W \qquad \bar{T} \in \{T_1,\ T_2, \cdots,\ T_k, \cdots,\ T_N\}$$

Decision Variables for Each Zone:

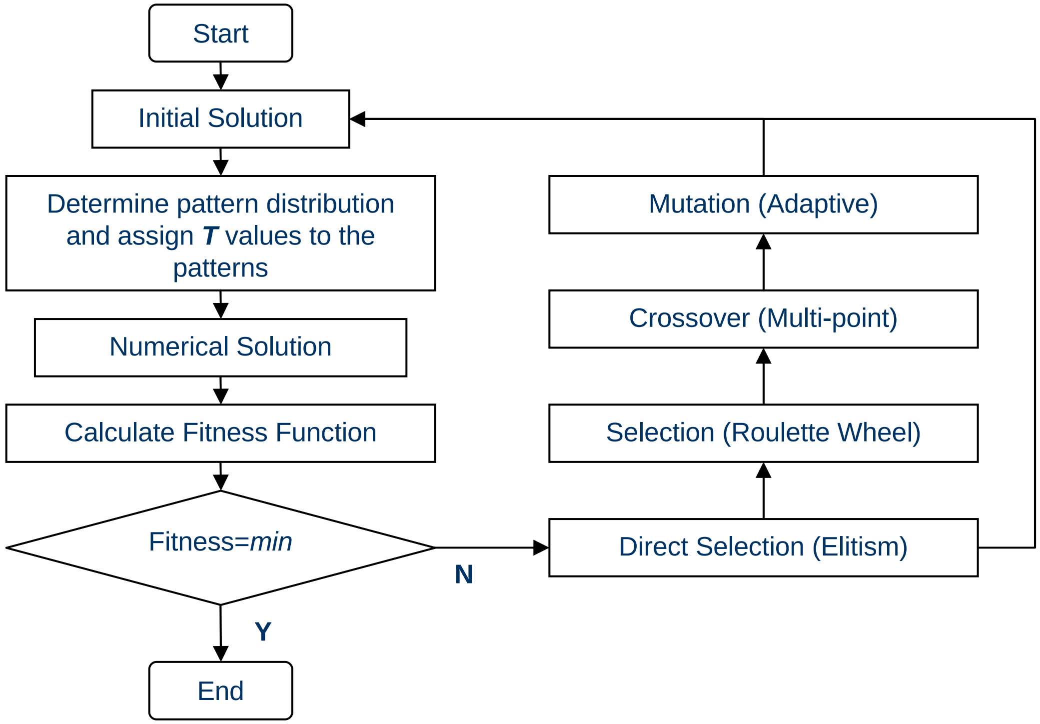
$$\bar{T}_j = T_k \ \text{ if } \ N_k = Max\{N_1,\ N_2, \cdots,\ N_k, \cdots,\ N_N\}$$

$$(T_i,\ \mu_{Xi},\ \mu_{Yi},\ \sigma_{Xi},\ \sigma_{Yi})$$

$$N(X, Y, \mu_{Xi}, \mu_{Yi}, \sigma_{Xi}, \sigma_{Yi}) = \frac{1}{2\pi\sigma_{Xi}\sigma_{Yi}}\exp\left\{-\frac{1}{2}\left[\left(\frac{X-\mu_{Xi}}{\sigma_{Xi}}\right)^2 + \left(\frac{Y-\mu_{Yi}}{\sigma_{Yi}}\right)^2\right]\right\}$$

Model Development

Problem Formulation and Search Procedure



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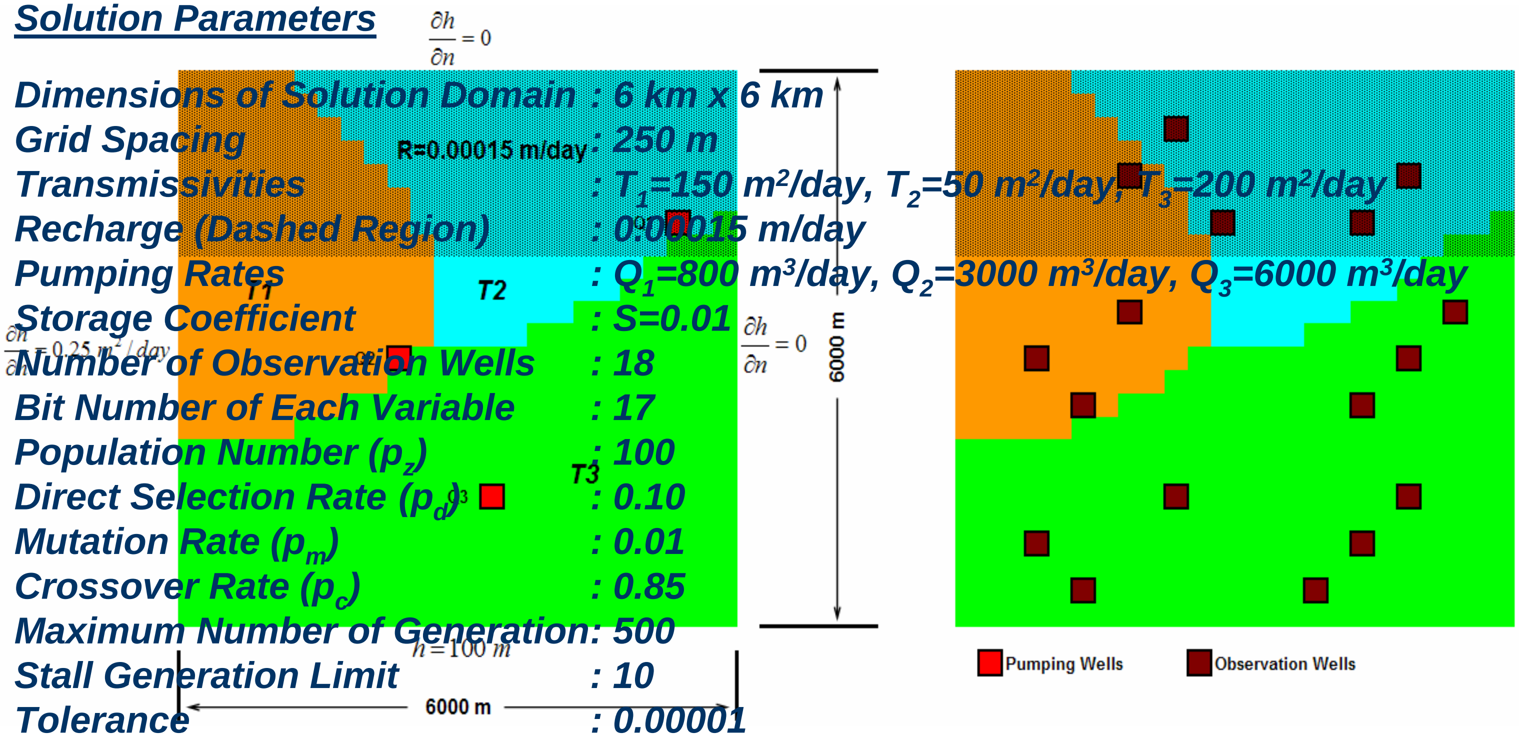
Numerical Example

Problem Definition

Solution Parameters

Dimensions of Solution Domain : 6 km x 6 km
Grid Spacing : 250 m
Transmissivities : T_1=150 m²/day, T_2=50 m²/day, T_3=200 m²/day
Recharge (Dashed Region) : 0.00015 m/day
Pumping Rates : Q_1=800 m³/day, Q_2=3000 m³/day, Q_3=6000 m³/day
Storage Coefficient : S=0.01
Number of Observation Wells : 18
Bit Number of Each Variable : 17
Population Number (p_z) : 100
Direct Selection Rate (p_d) : 0.10
Mutation Rate (p_m) : 0.01
Crossover Rate (p_c) : 0.85
Maximum Number of Generation: 500
Stall Generation Limit : 10
Tolerance : 0.00001



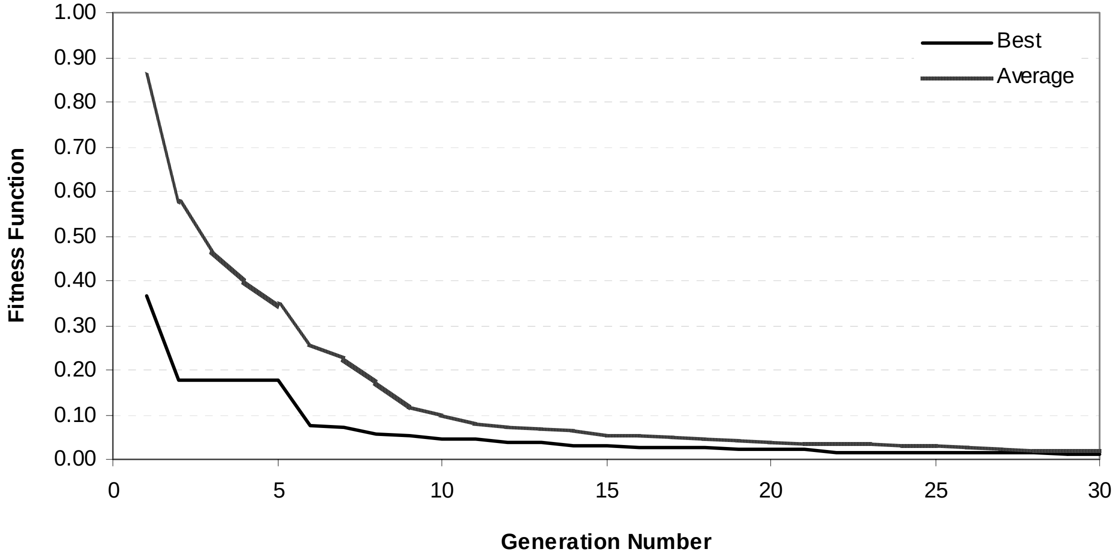
Numerical Example

Three cases have been taken into account to test the performance of the proposed model

- Case 1 (Pattern known – Parameters unknown)
- Case 2 (Pattern unknown – Parameters known)
- Case 3 (Pattern unknown – Parameters unknown)

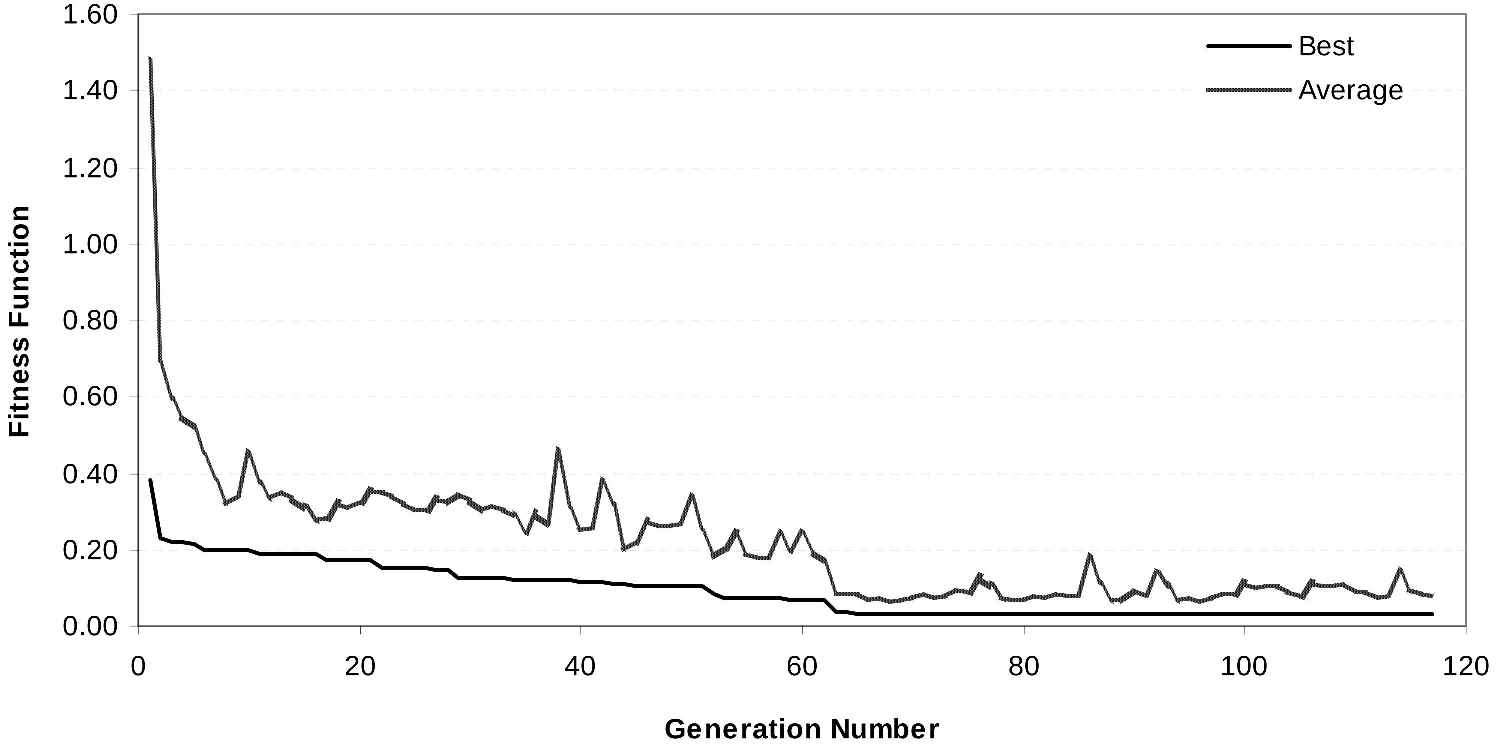
Numerical Example

Case 1 (Pattern known – Parameters unknown)



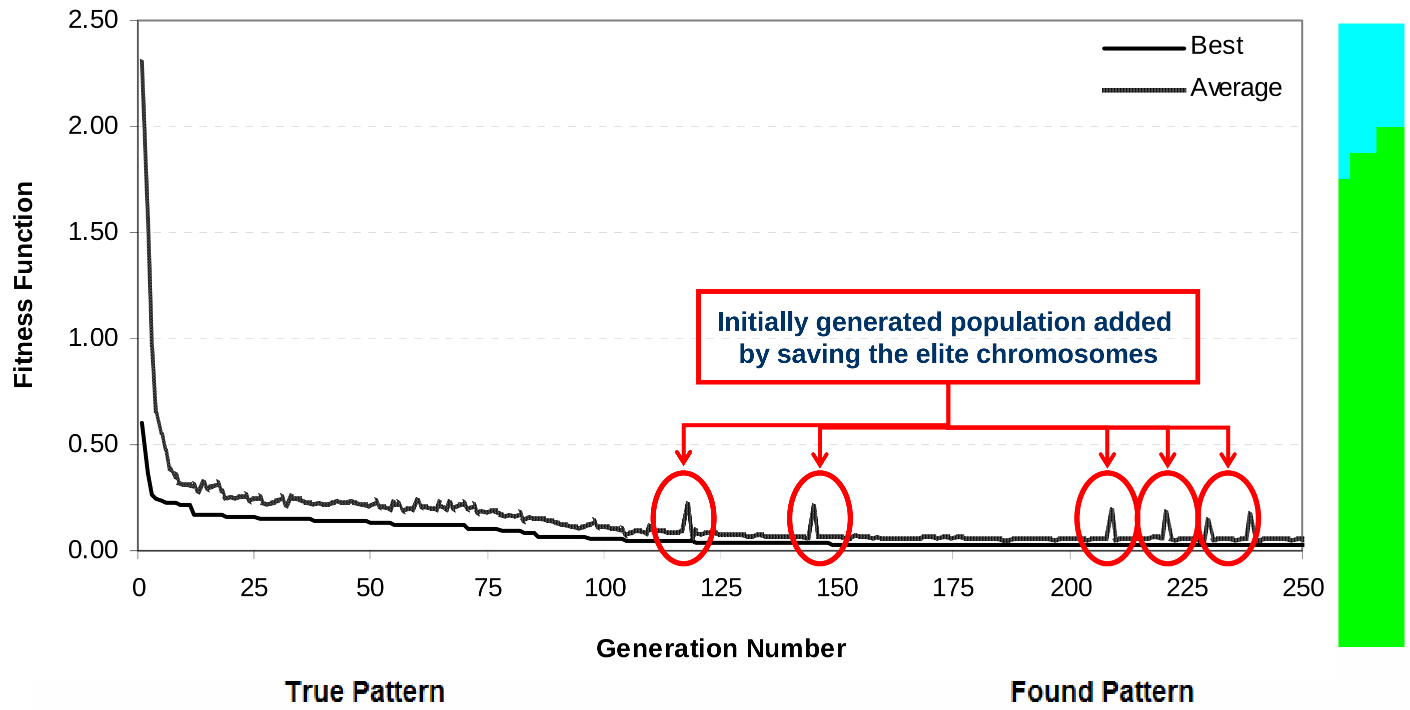
Numerical Example

Case 2 (Pattern unknown – Parameters known)



Numerical Example

Case 3 (Pattern unknown – Parameters unknown)



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Conclusions

Following conclusions can be drawn from this study:

- The proposed solution algorithm is applied to a hypothetical aquifer model to determine the transmissivity values and their patterns.

- While determination of aquifer parameters (inverse problem) is relatively easy, on the other hand detemination of parameter zonations is more difficult.

- The value of the objective function is improved quickly in early iterations and then changes very slowly in subsequent iterations.

- Many neighbor solutions, which have similar values of decision variables, result in the same zonation and thus cause slow improvement of the value of objective function

Conclusions

Following conclusions can be drawn from this study:

- One of the major difficulties to find the global optimum solution is the loosing the diversity of the population pool.

- To satisfy the diversity of the population, initial population terms have been added to the pool by checking the stall generation limit.

- Results showed that after satisfying the diversity of population, variation of the fitness function increares.

- Model may be trapped to local optimum solutions when the number of population is small.

- After building the generalized solution technique, future study should be done on a real aquifer.

Thank you…
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