



Uncertainty Modeling in Health Risk Assessment and Groundwater Resources Management

Elcin Kentel

Dr. Mustafa M. Aral

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Probabilistic-fuzzy health risk modeling

- Health risk analysis of multi-pathway exposure to contaminated water

uncertain parameters

variable parameters

not all uncertainties
are due to randomness

fuzzy set theory

currently treated with
statistical approaches

- ❖ scarce or incomplete data
- ❖ measurement errors
- ❖ data obtained from expert judgement
- ❖ subjective interpretation of available info.

- ❖ randomness

Human health risk assessment – simultaneous probabilistic & possibilistic uncertainty propagation

Probabilistic-fuzzy health risk modeling

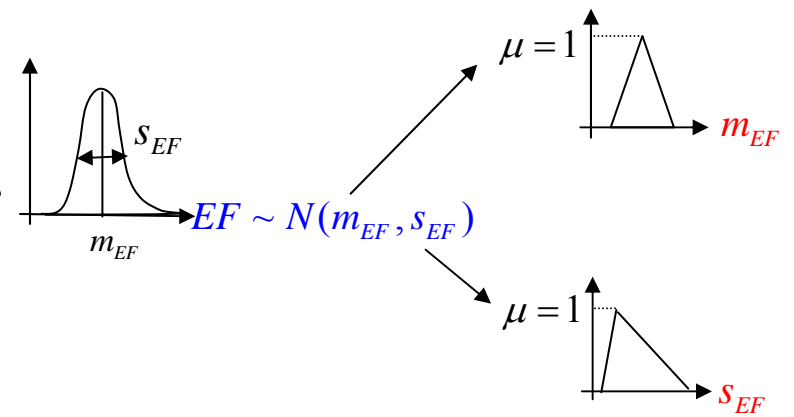
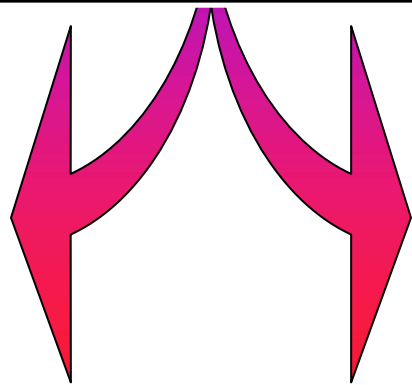
2D MC vs 2D Fuzzy MC health risk assessment

$$Risk = \sum_{r=ing.,inh.,der.} Risk_r = \sum_{r=ing.,inh.,der.} CDI_r \cdot CPF_r$$

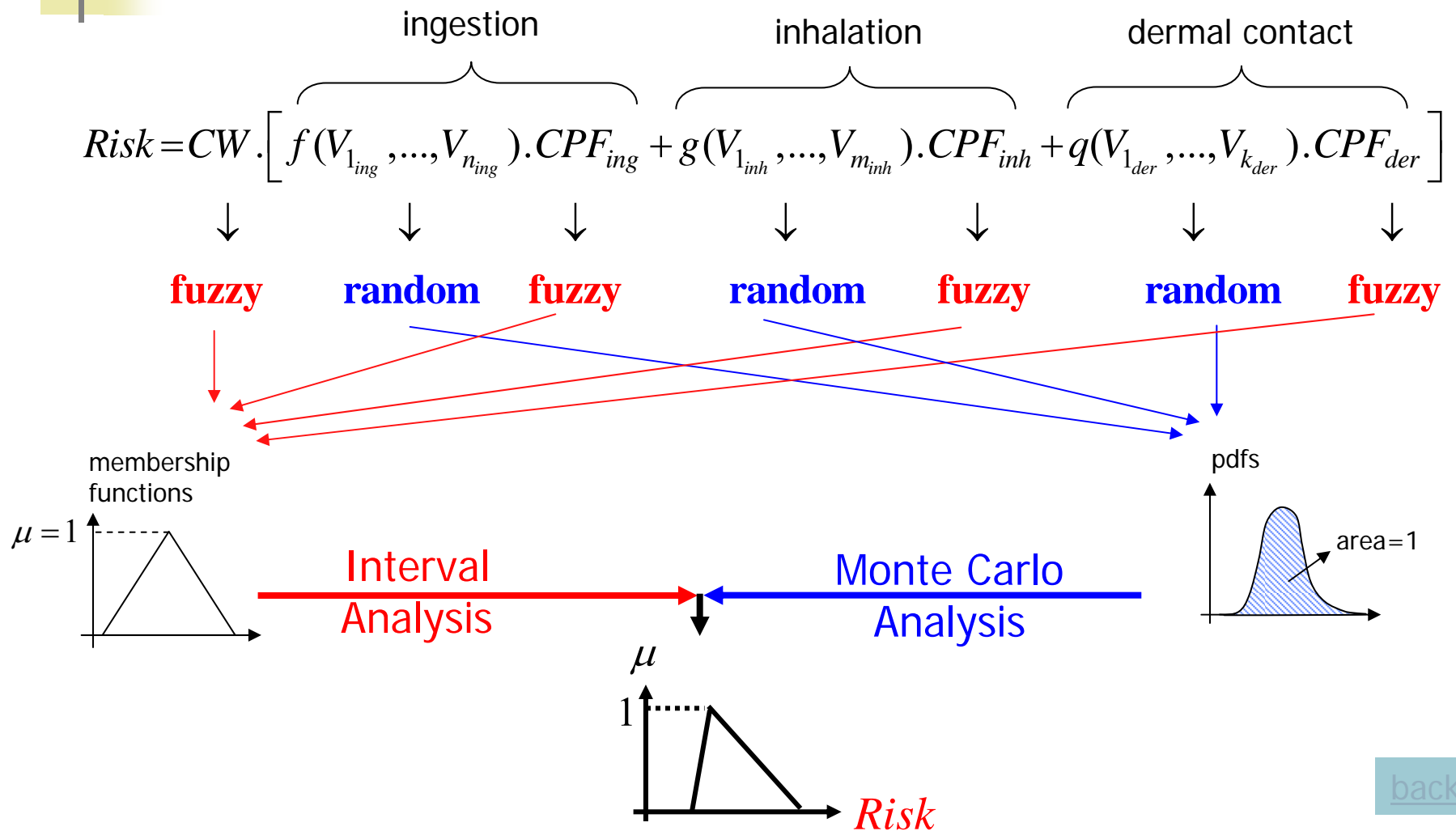
$$CDI_r = C_r \frac{IR_r \cdot EF \cdot ED}{BW \cdot AT} \quad CDI_r = C_r \frac{SA \cdot F \cdot PC \cdot ET \cdot EF \cdot ED \cdot CF}{BW \cdot AT}$$

$$Risk = C \cdot \left[f(V_{1_{ing}}, \dots, V_{n_{ing}}) \cdot CPF_{ing} + g(V_{1_{inh}}, \dots, V_{m_{inh}}) \cdot CPF_{inh} + q(V_{1_{der}}, \dots, V_{k_{der}}) \cdot CPF_{der} \right]$$

fuzzy random fuzzy



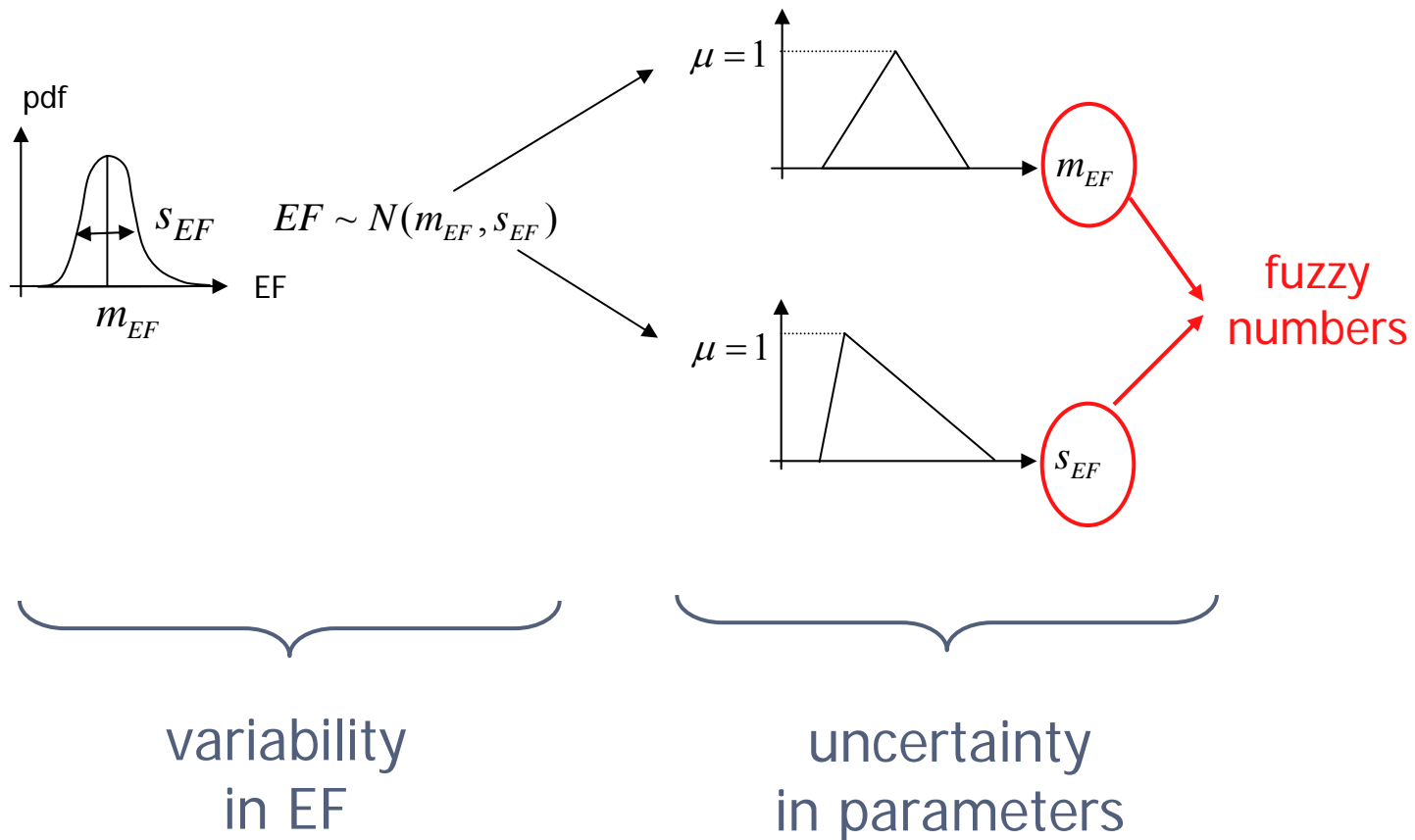
Probabilistic-fuzzy health risk modeling



2D Fuzzy Monte Carlo

$$CDI = \frac{C \cdot IR \cdot EF \cdot ED}{BW \cdot AT}$$

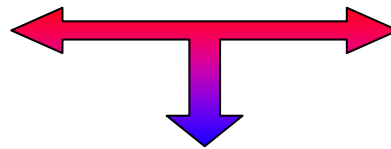
$$Risk = CDI \cdot CPF$$



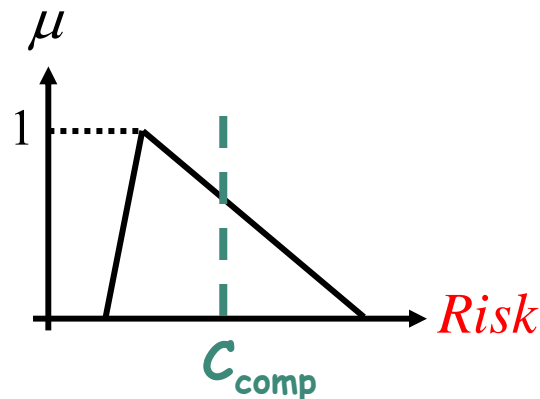
Human health risk assessment –

simultaneous probabilistic & possibilistic uncertainty propagation

Probabilistic-fuzzy
health risk modeling



2D MC versus 2D Fuzzy MC
health risk assessment

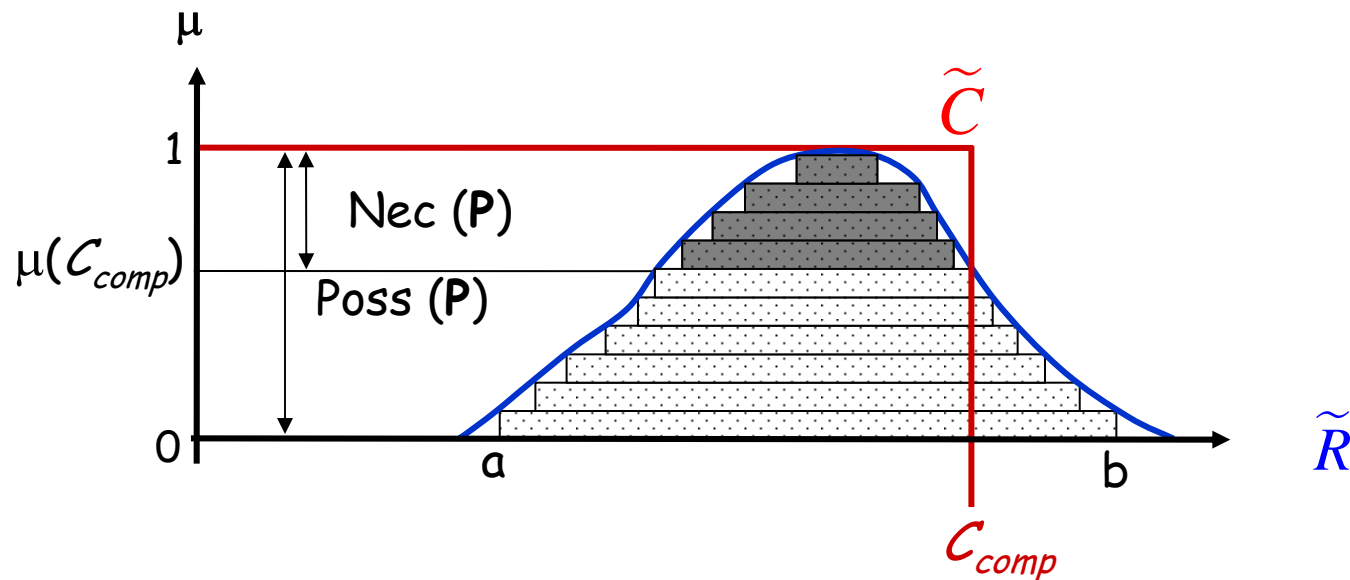


Is the fuzzy risk
acceptable
with respect to
this compliance criterion ?

or
→
validity of P

P: "the fuzzy risk, R is
smaller than or equal
to the compliance
criterion, C_{comp} "

The Possibility and The Necessity Measures



Any evidence that supports **P** \rightarrow Poss (P) = 1
(dotted sections)

Evidence that supports impossibility of **not P** \rightarrow Nec (P) = $1 - \mu(c_{comp})$
(shaded sections)



Currently recommended

- Comé et al. (1997) suggested → possibility measure
necessity measure

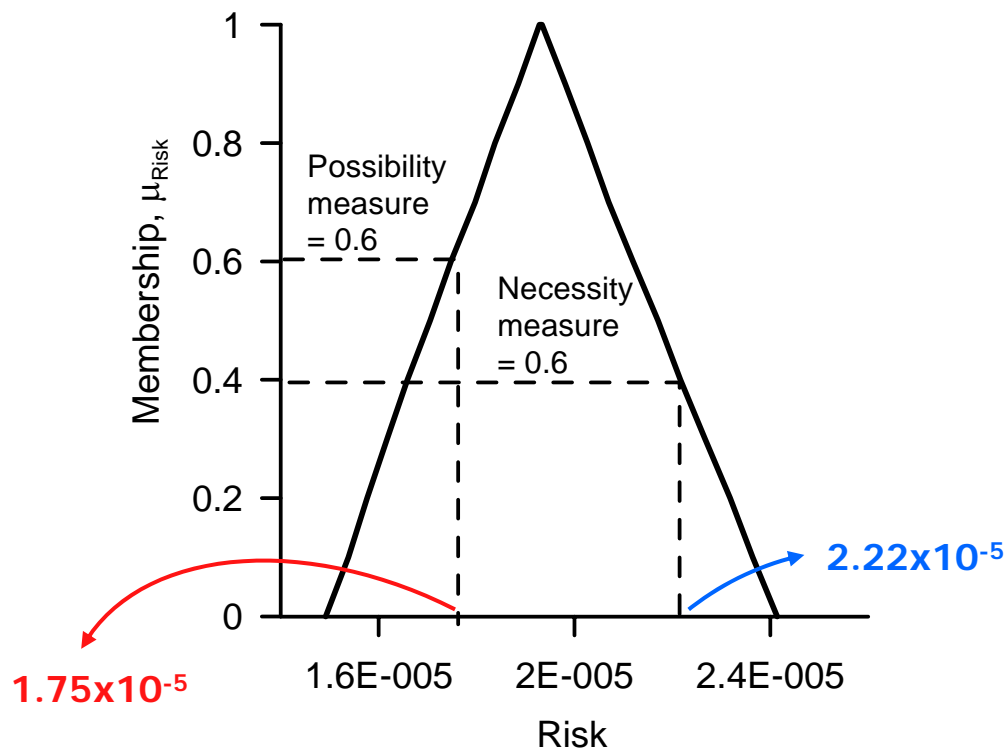
↑ conservative
Human health risk
assessment

- Only necessity measure → some valuable
information lost
↓ more informative

Combined measure

Compliance for Possibilistic RA

- ❖ Guideline: the fuzzy risk R should not exceed 2.0×10^{-5} for a possibility/necessity measure of 0.6



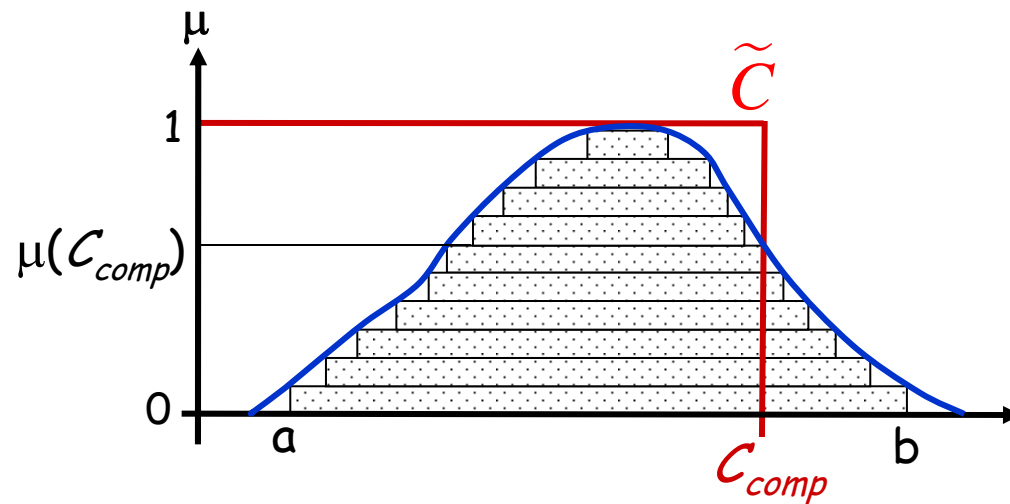
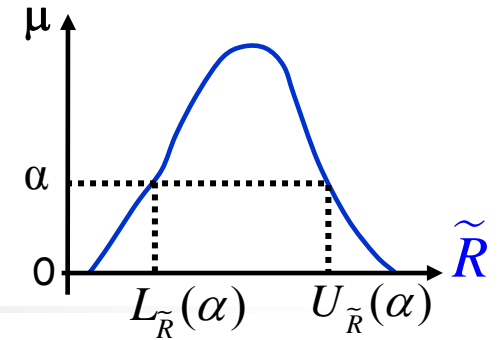
The necessity measure of a proposition is always smaller than its possibility measure

risk corresponding to a necessity

The necessity measure is conservative
NOT acceptable

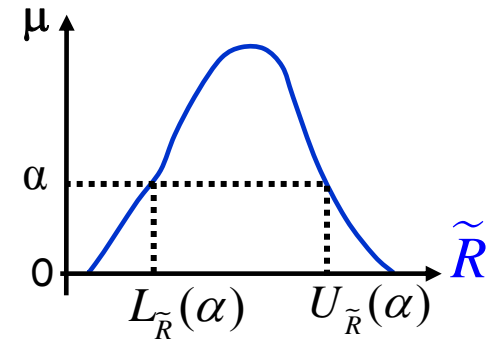
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The risk tolerance measure



$$T(\mathbf{P}) = \begin{cases} 0 & C_{comp} \leq L_{\tilde{R}}(0) \\ \frac{1}{2}(\beta Poss(\mathbf{P}) + \gamma Nec(\mathbf{P})) & L_{\tilde{R}}(0) < C_{comp} < U_{\tilde{R}}(0) \\ 1 & C_{comp} \geq U_{\tilde{R}}(0) \end{cases}$$

The risk tolerance measure

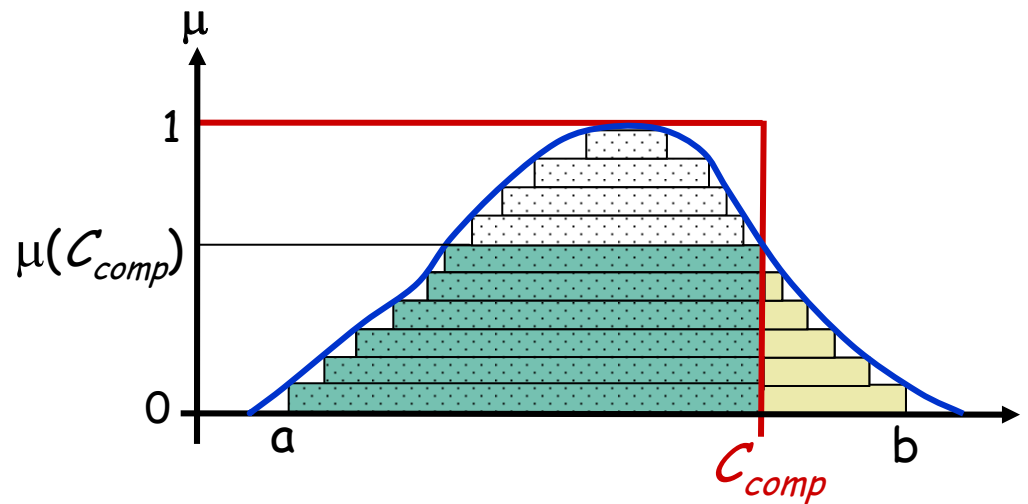


$$T(\mathbf{P}) = \begin{cases} 0 & C_{comp} \leq L_{\tilde{R}}(0) \\ \frac{1}{2} (\beta Poss(\mathbf{P}) + \gamma Nec(\mathbf{P})) & L_{\tilde{R}}(0) < C_{comp} < U_{\tilde{R}}(0) \\ 1 & C_{comp} \geq U_{\tilde{R}}(0) \end{cases}$$

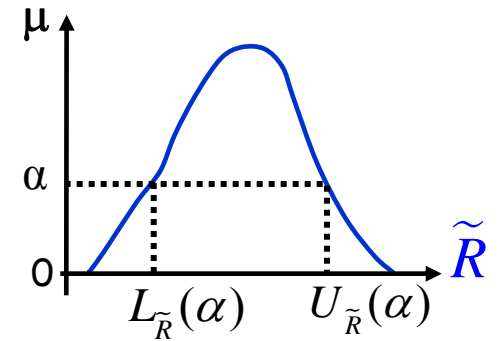
$$\beta = \frac{A_{poss_l}}{A_{poss_T}}$$

$$A_{poss_l} = \int_0^{\mu_{\tilde{R}}(C_{comp})} \alpha [C_{comp} - L_{\tilde{R}}(\alpha)] d\alpha$$

$$A_{poss_T} = \int_0^{\mu_{\tilde{R}}(C_{comp})} \alpha [U_{\tilde{R}}(\alpha) - L_{\tilde{R}}(\alpha)] d\alpha$$



The risk tolerance measure



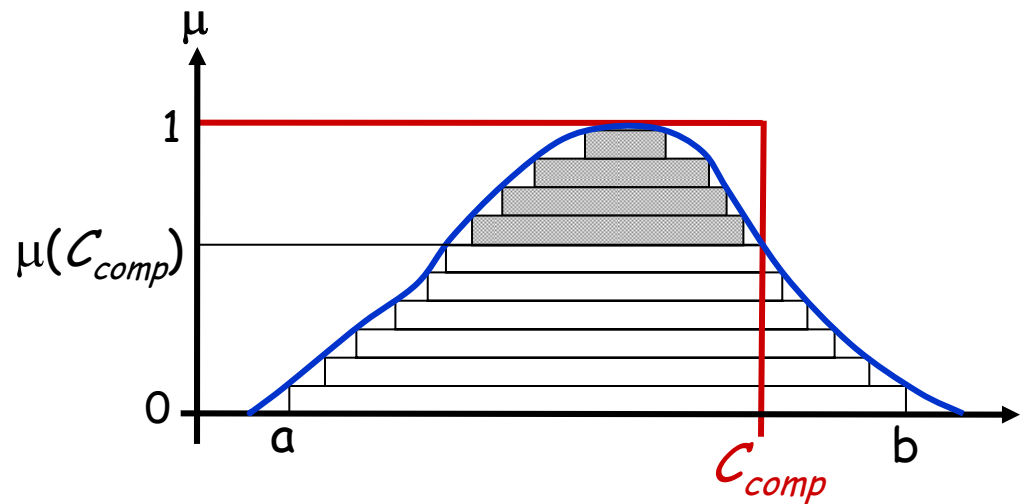
$$T(\mathbf{P}) = \begin{cases} 0 & C_{comp} \leq L_{\tilde{R}}(0) \\ \frac{1}{2}(\beta Poss(\mathbf{P}) + \gamma Nec(\mathbf{P})) & L_{\tilde{R}}(0) < C_{comp} < U_{\tilde{R}}(0) \\ 1 & C_{comp} \geq U_{\tilde{R}}(0) \end{cases}$$

$$\gamma = \frac{A_{nec_l}}{A_{nec_T}}$$

$$A_{nec_l} = \int_0^{\mu_{\tilde{R}}(C_{comp})} \alpha [C_{comp} - L_{\tilde{R}}(\alpha)] d\alpha$$

$$A_{nec_T} = \int_0^{\mu_{\tilde{R}}(C_{comp})} \alpha [U_{\tilde{R}}(\alpha) - L_{\tilde{R}}(\alpha)] d\alpha$$

both areas are defined for $U_{\tilde{R}}(\mu_{\tilde{R}}(C_{comp})) = C_{comp}$



Coefficient for Necessity

$$\gamma = \frac{A_{nec_l}}{A_{nec_T}}$$

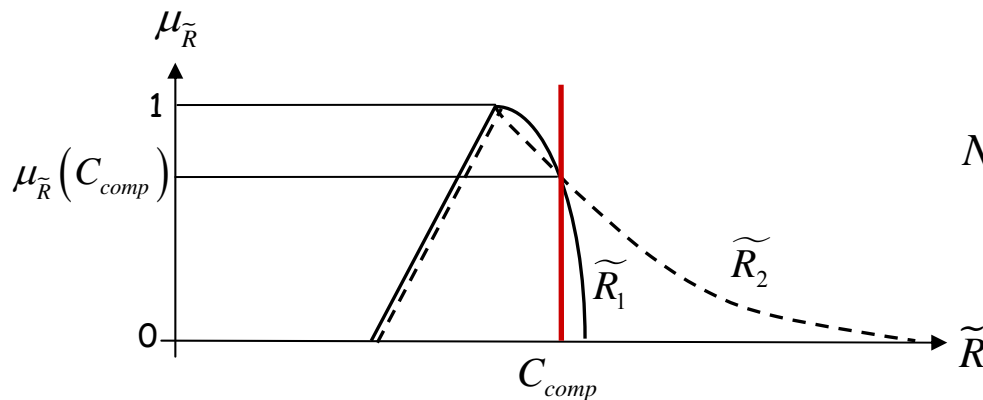
$$A_{nec_l} = \begin{cases} 0 \\ \int_0^{\mu_{\tilde{R}}(C_{comp})} \alpha [C_{comp} - L_{\tilde{R}}(\alpha)] d\alpha \end{cases}$$

$$A_{nec_T} = \int_0^{\mu_{\tilde{R}}(C_{comp})} \alpha [U_{\tilde{R}}(\alpha) - L_{\tilde{R}}(\alpha)] d\alpha$$

$$L_{\tilde{R}}(M_{\tilde{R}}(C_{comp})) = C_{comp}$$

$$U_{\tilde{R}}(M_{\tilde{R}}(C_{comp})) = C_{comp}$$

Effect of membership function on the measures

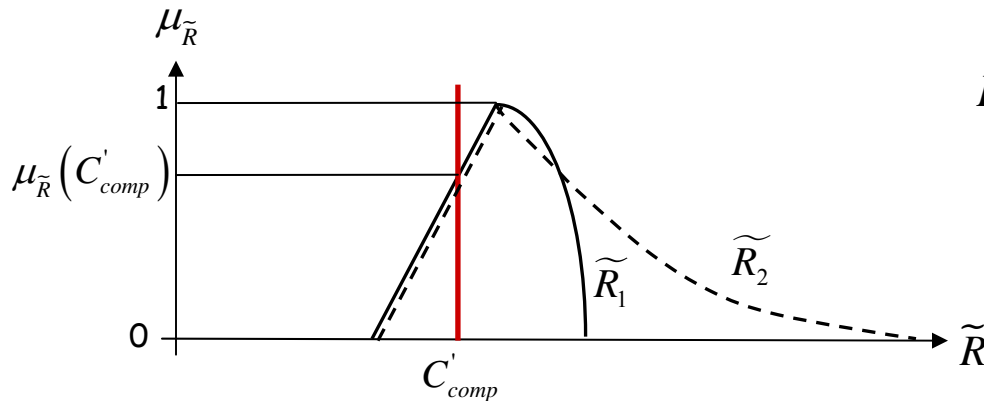


$$Poss(\tilde{R}_1 \leq \tilde{C}) = Poss(\tilde{R}_2 \leq \tilde{C}) = 1$$

Degree of compliance of \tilde{R}_1 is greater than that of \tilde{R}_2

$$Nec(\tilde{R}_1 \leq \tilde{C}) = Nec(\tilde{R}_2 \leq \tilde{C}) = 1 - \mu_{\tilde{R}_1}(C_{comp})$$

$$T(\tilde{R}_1 \leq \tilde{C}) > T(\tilde{R}_2 \leq \tilde{C})$$



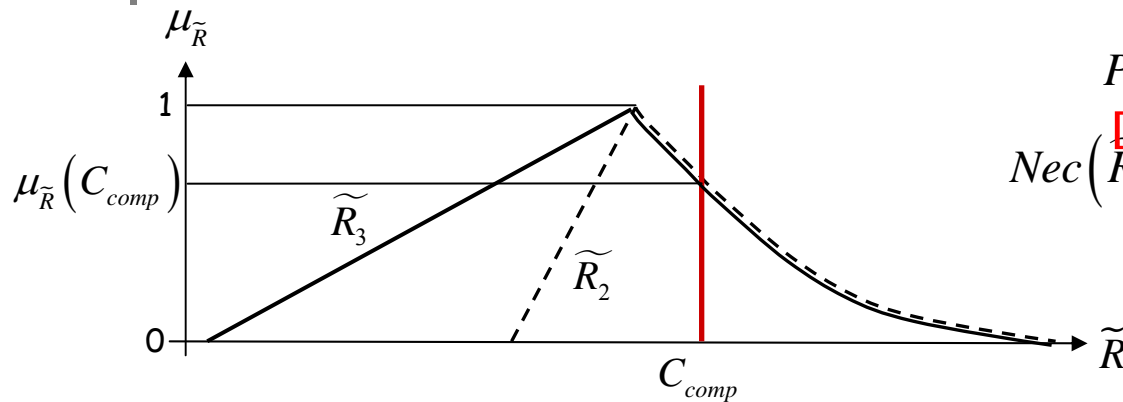
$$Poss(\tilde{R}_1 \leq \tilde{C}') = Poss(\tilde{R}_2 \leq \tilde{C}') = \mu_{\tilde{R}_1}(C'_{comp})$$

Degree of compliance of \tilde{R}_1 is greater than that of \tilde{R}_2

$$Nec(\tilde{R}_1 \leq \tilde{C}') = Nec(\tilde{R}_2 \leq \tilde{C}') = 0$$

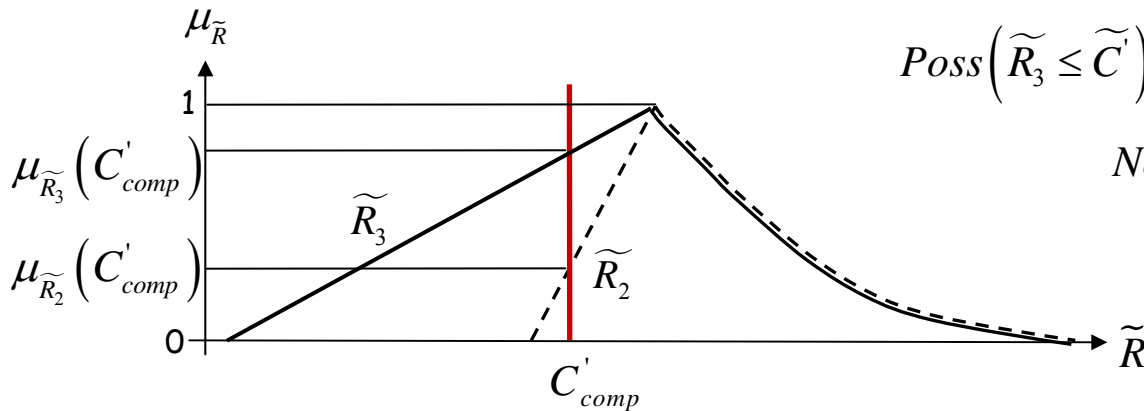
$$T(\tilde{R}_1 \leq \tilde{C}') > T(\tilde{R}_2 \leq \tilde{C}')$$

Effect of membership function on the measures



$$\begin{aligned}
 Poss(\tilde{R}_3 \leq \tilde{C}) &= Poss(\tilde{R}_2 \leq \tilde{C}) = 1 \\
 Nec(\tilde{R}_3 \leq \tilde{C}) &= Nec(\tilde{R}_2 \leq \tilde{C}) = 1 - \mu_{\tilde{R}_3}(C_{comp}) \\
 T(\tilde{R}_3 \leq \tilde{C}) &> T(\tilde{R}_2 \leq \tilde{C})
 \end{aligned}$$

Degree of compliance of \tilde{R}_3 is greater than that of \tilde{R}_2

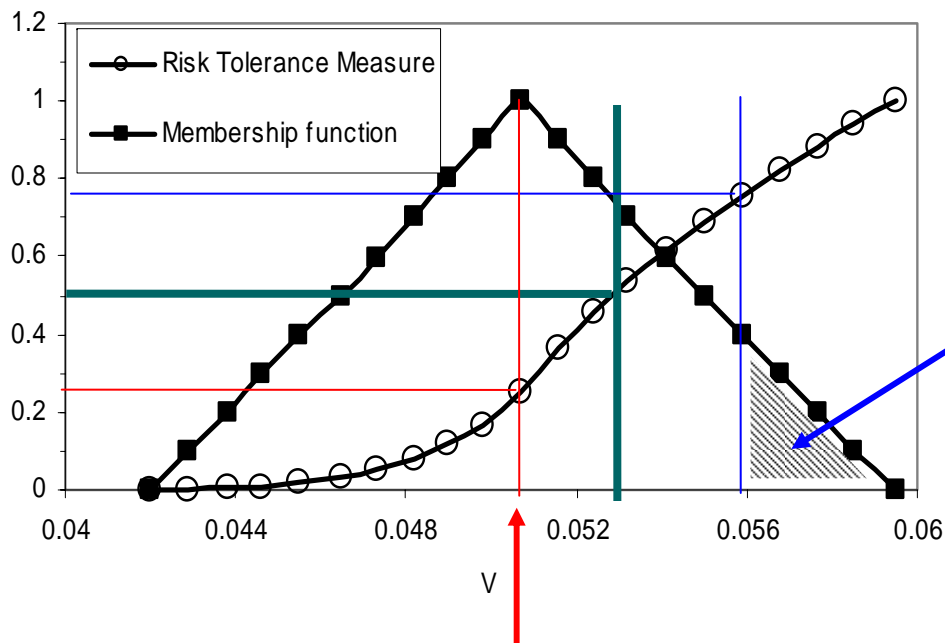


$$\begin{aligned}
 Poss(\tilde{R}_3 \leq \tilde{C}') &= \mu_{\tilde{R}_3}(C'_{comp}) > Poss(\tilde{R}_2 \leq \tilde{C}') = \mu_{\tilde{R}_2}(C'_{comp}) \\
 Nec(\tilde{R}_3 \leq \tilde{C}') &= Nec(\tilde{R}_2 \leq \tilde{C}') = 0 \\
 T(\tilde{R}_3 \leq \tilde{C}') &> T(\tilde{R}_2 \leq \tilde{C}')
 \end{aligned}$$

Degree of compliance of \tilde{R}_3 is greater than that of \tilde{R}_2

Reasonable Minimum Risk Tolerance Value

$\mu_{\tilde{v}}$, Risk Tolerance



If the design criteria is here the $RT=0.25$

To have a RT of 0.75 only this shaded region is allowed to lie on the right of the design criteria

A minimum risk tolerance value of 0.5 seems reasonable



Conclusions for Risk Tolerance Measure

- ➡ To make decisions about compliance of a **fuzzy risk** with respect to a **crisp guideline**
- ➡ Include **all available information**: possibility measure & necessity measure
- ➡ Establishment of a standard procedure needs careful examination of the results of **real case** possibilistic and hybrid risk assessment studies. For ex., necessity measure might be preferable for decision making in a school district.