Global Temperature Change and Sea-Level Rise

M. M. Aral, J. Guan and B. Chang
Multimedia Environmental Simulations Laboratory
School of Civil and Environmental Engineering
Georgia Institute of Technology
Global Temperature Change:
Global Sea-Level Rise:

![Graph showing sea-level rise over time from 1880 to 2000]
Correlation:

- Graph showing the rate of sea level change (mm/year) vs. warming above 1951-1980 mean.
### What does IPCC say?

The _Summary for Policy Makers_ (SPM) released recently provides the following table of sea level rise projections:

<table>
<thead>
<tr>
<th>Case</th>
<th>Sea Level Rise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m at 2090-2099 relative to 1980-1999)</td>
</tr>
<tr>
<td></td>
<td>Model-based range</td>
</tr>
<tr>
<td></td>
<td>excluding future rapid dynamical changes in ice flow</td>
</tr>
<tr>
<td>B1 scenario</td>
<td>0.18 – 0.38</td>
</tr>
<tr>
<td>A1T scenario</td>
<td>0.20 – 0.45</td>
</tr>
<tr>
<td>B2 scenario</td>
<td>0.20 – 0.43</td>
</tr>
<tr>
<td>A1B scenario</td>
<td>0.21 – 0.48</td>
</tr>
<tr>
<td>A2 scenario</td>
<td>0.23 – 0.51</td>
</tr>
<tr>
<td>A1FI scenario</td>
<td>0.26 – 0.59</td>
</tr>
</tbody>
</table>
What does IPCC say?
Semi-empirical models:

Rahmstorf’s Study (Science, Vol. 315 pp.19, 2007)

\[
\frac{\partial H}{\partial t} = a \left( T - T_o \right)
\]

\[
H(t) = a \int_{t_o}^{t} \left( T(\tau) - T_o \right) d\tau
\]
Semi-empirical models:

Rahmstorf’s Study  (Science, Vol. 315 pp.19, 2007)
Semi-empirical models:

Rahmstorf’s Study  (Science, Vol. 315 pp.19, 2007)

Sea-Level Rise Interval Predicted:

\[ \{0.5 - 1.4m\} \] above 1990 level

IPCC Interval Predicted:

\[ \{0.09 - 0.88m\} \] above 1990 level
IPCC and other Models:
Everything should be made as simple as possible, but not simpler.

A. Einstein
A Dynamic Systems Model
Earlier studies showed that the relationship between $T$ & $H$ is linear.

Hypothesis:

- **Temperature:** $T = f_1(T, H, U, c_1)$
- **Sea-Level:** $H = f_2(T, H, U, c_2)$
Proposed Model:

\[
\frac{dT(t)}{dt} = a_{11} T(t) + a_{12} H(t) + a_{13} U_i(t) + c_1
\]

\[
\frac{dH(t)}{dt} = a_{21} T(t) + a_{22} H(t) + a_{23} U_i(t) + c_2
\]
Proposed Model: (Simplified)

\[
\frac{dT(t)}{dt} = a_{11} T(t) + a_{12} H(t) + c_1
\]

\[
\frac{dH(t)}{dt} = a_{21} T(t) + a_{22} H(t) + c_2
\]
Proposed Model:

\[
X(t) = \left(T(t), H(t)\right) = \left(x_1(t), x_2(t)\right)
\]

\[
A = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\]

\[
C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}
\]

\[
\frac{dX}{dt} = AX(t) + C
\]
Proposed Discrete Model:

\[ X(\Phi X 1) = \Omega (k) + \]

\[ \Phi = (I + A\Delta t); \quad \Omega = C\Delta t \]
LSM Model to determine $a_{ij}$ & $c_i$: 

$$F^* = \minimize_{\phi_i} \left\{ \left( Y_i \Lambda \phi \right) \tilde{Y} \left( \begin{array}{c} i \\ \Lambda \phi \end{array} \right) \right\}$$
Confidence Interval:

\[ \hat{T}_{CI}(k) = \hat{T}(k) \pm t_{\alpha/2, n-4} \sqrt{\hat{\sigma}_T^2 \left( 1 + \hat{Z}(k)^\tau \left( \Lambda^\tau \Lambda \right)^{-1} \hat{Z}(k) \right)} \]

\[ \hat{H}_{CI}(k) = \hat{H}(k) \pm t_{\alpha/2, n-4} \sqrt{\hat{\sigma}_H^2 \left( 1 + \hat{Z}(k)^\tau \left( \Lambda^\tau \Lambda \right)^{-1} \hat{Z}(k) \right)} \]
Application:

- 2-year moving average outcome is used for both state variables.
Temperature Data:

![Graph showing temperature data over time with moving averages.](image)
Sea-Level Data:

- Original data
- 2-year moving average
- 5-year moving average
### Matrix Coefficients:

<table>
<thead>
<tr>
<th>Data used</th>
<th>Discrete system (Φ, Ω)</th>
<th>Continuous system (A, C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880 - 1950</td>
<td>$\begin{bmatrix} 0.8337 &amp; 0.0072 \ 0.3441 &amp; 0.9960 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -0.1663 &amp; 0.0072 \ 0.3441 &amp; -0.0040 \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>$\begin{bmatrix} 0.1663 \ 0.0083 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.0083 \ 0.2234 \end{bmatrix}$</td>
</tr>
<tr>
<td>1880 - 2001</td>
<td>$\begin{bmatrix} 0.8074 &amp; 0.0068 \ 0.4115 &amp; 0.9956 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -0.1926 &amp; 0.0068 \ 0.4115 &amp; -0.0045 \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>$\begin{bmatrix} -0.0110 \ 0.2585 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -0.0110 \ 0.2585 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
Temperature (1880-1950): (90%) Conf. Int.

\[ R^2 = 0.6 \]
Sea-Level (1880-1950): (90%) Conf. Int.

R² = 0.8

![Graph showing temperature changes from 1880 to 2000 with original data, prediction, and confidence interval. The graph includes a line indicating the prediction and a shaded area for the confidence interval. The R² value is 0.5.](image)

R² = 0.5

\[ R^2 = 0.9 \]
Temperature (1990-2100)
with IPCC scenarios:
Sea-Level (1990-2100) with IPCC scenarios:

- Original data
- Prediction
- Confidence interval
- Rahmstorf's results
- IPCC interval

Sea-level (cm)

(b)

1880 1900 1920 1940 1960 1980 2000 2020 2040 2060 2080 2100
Sea-Level (1990-2100) with IPCC scenarios:

- Original data
- Prediction
- IPCC interval
- Rahmstorf's interval


Sea-level (cm): -20, 0, 20, 40, 60, 80, 100, 120, 140

Graph showing sea-level changes over time with different scenarios.

- Solid line: IPCC interval
- Dotted line: Rahmstorf's interval
- Dashed line: Prediction
- Black dots: Original data