



Contaminant Source Identification in River Networks

A Probabilistic Analysis and Application to Altamaha River

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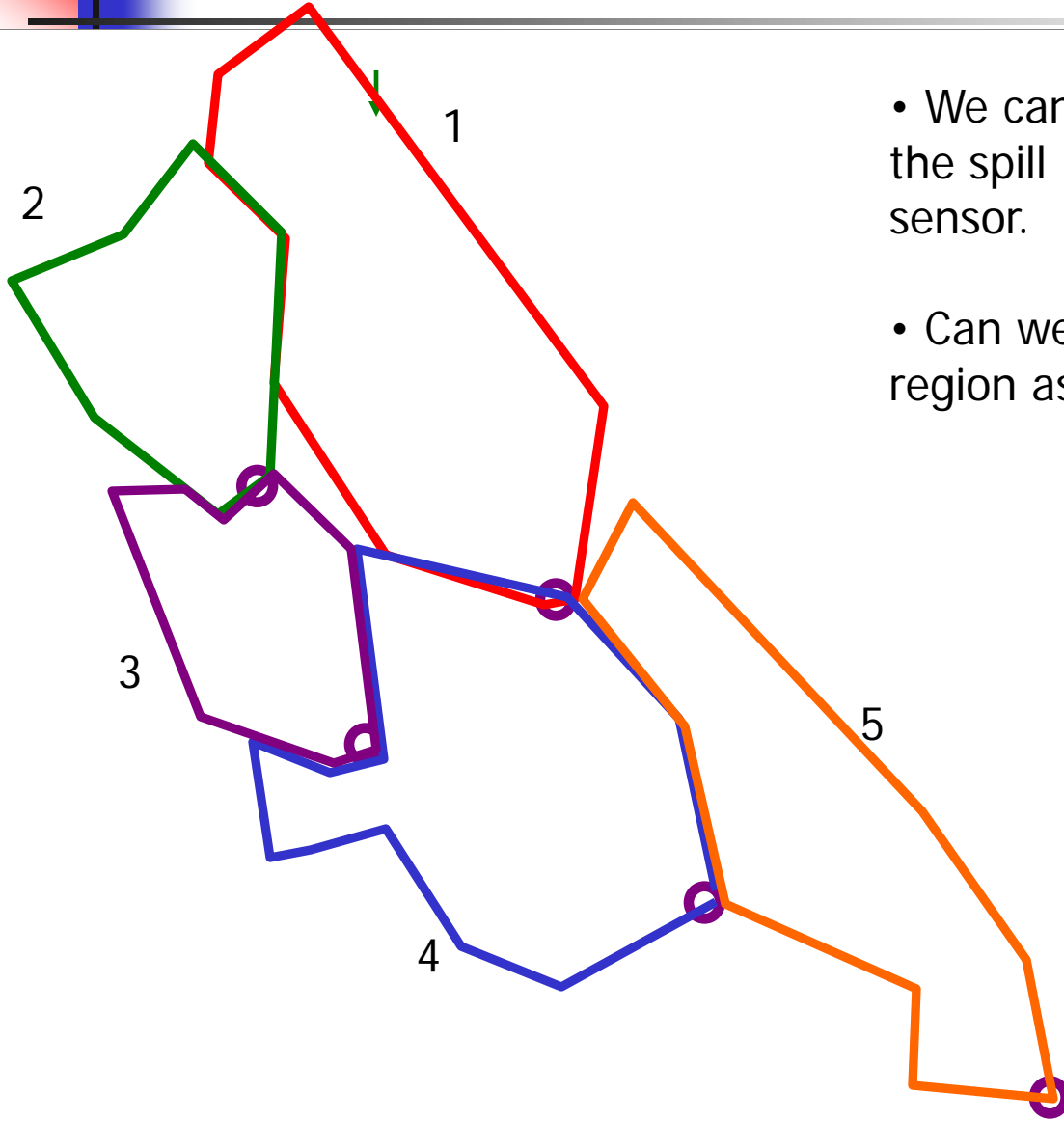
Problem



Given the monitoring network, can we determine the location of the spill?



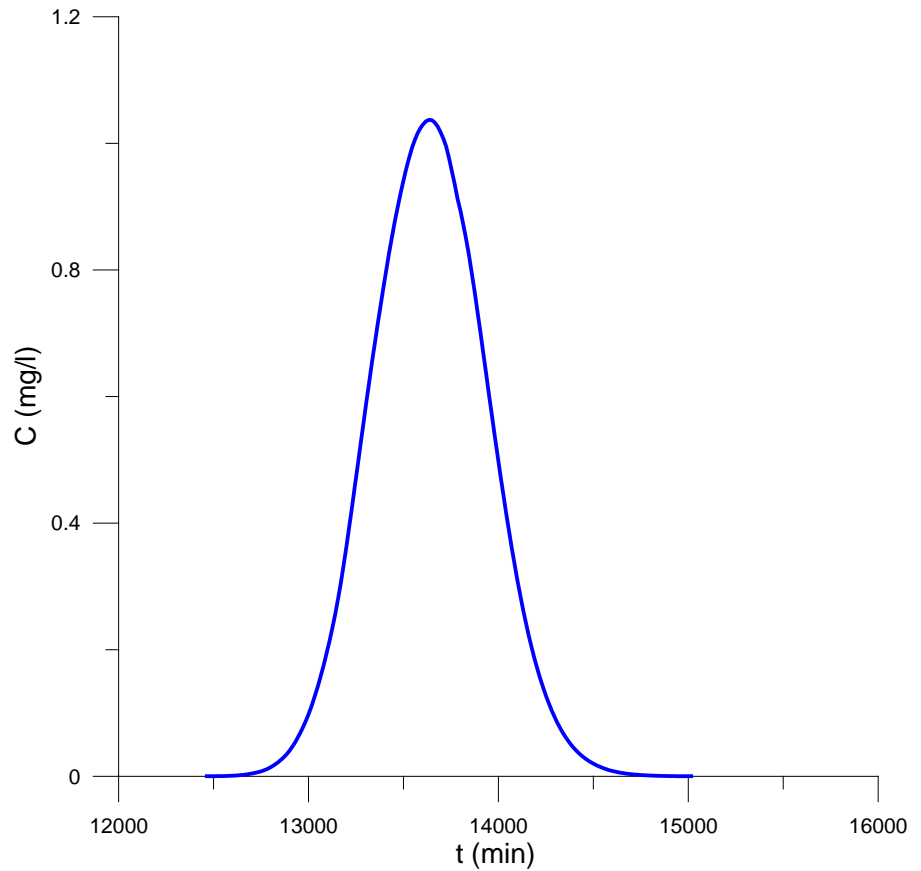
Problem



- We can easily determine a region for the spill if we know the first activated sensor.
- Can we find a set of junctions in that region as possible spill locations?



Spill Example

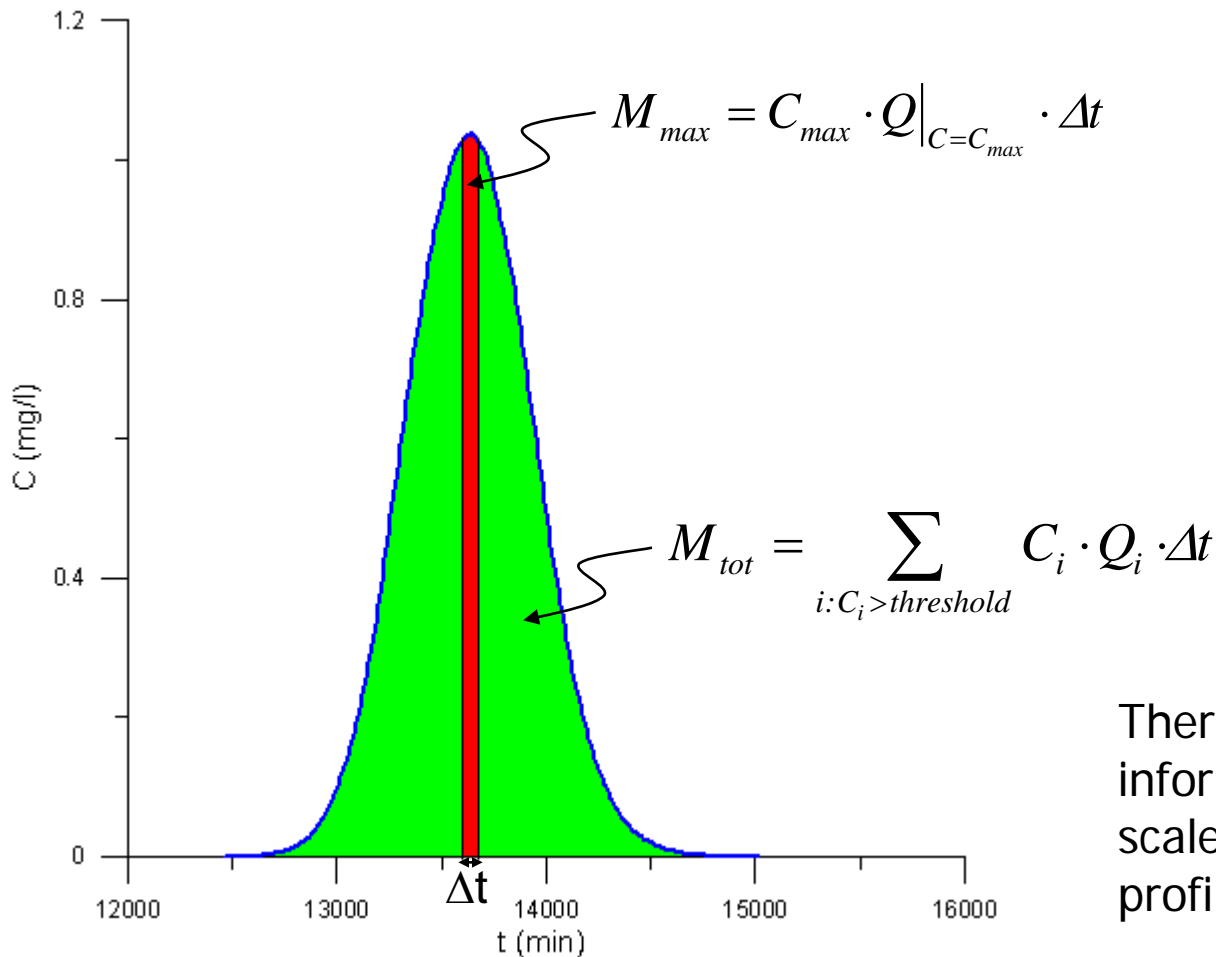


Concentration Profile measured at J9



Parameterization of Concentration Profile

Parameter 1: M_{max}/M_{tot}



$$\frac{M_{max}}{M_{tot}} = \frac{C_{max} \cdot Q|_{C=C_{max}}}{\sum_{i:C_i > threshold} C_i \cdot Q_i}$$

Assuming steady flow:

$$\frac{M_{max}}{M_{tot}} = \frac{C_{max}}{\sum_{i:C_i > threshold} C_i}$$

Therefore, this parameter gives information about the vertical scale of the concentration profile.



Parameterization of Concentration Profile

Parameter 2: Peclet Number

$$Pe = \frac{\left(\sum_{k:C_k > \text{threshold}} \frac{1}{v_k} v_k \Delta t \right)^2}{\sum_{k:C_k > \text{threshold}} \frac{D_k}{v_k^3} v_k \Delta t}$$

$$D = 0.011 \frac{v^2 B^2}{dU_*} \quad (\text{Fisher et al. 1979})$$

$$Pe = \frac{\left(\sum_{k:C_k > \text{threshold}} \Delta t \right)^2}{\sum_{k:C_k > \text{threshold}} 0.011 \left(\frac{B^2}{dU_*} \right)_k \Delta t}$$

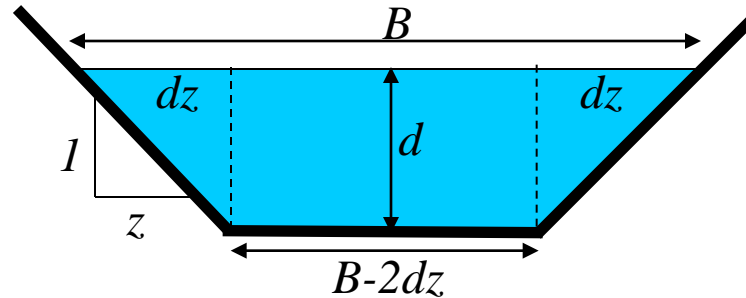
v : velocity

D : Dispersion coefficient

B : Top width

d : depth of flow

U_* : Shear velocity



$$\frac{B^2}{dU_*} = \frac{B^2}{d\sqrt{gRS_o}}$$

$$\frac{B^2}{dU_*} = \frac{B^2}{d\sqrt{g\left(\frac{Bd - d^2z}{2\sqrt{d^2z^2 + d^2} + B - 2dz}\right)S_o}}$$

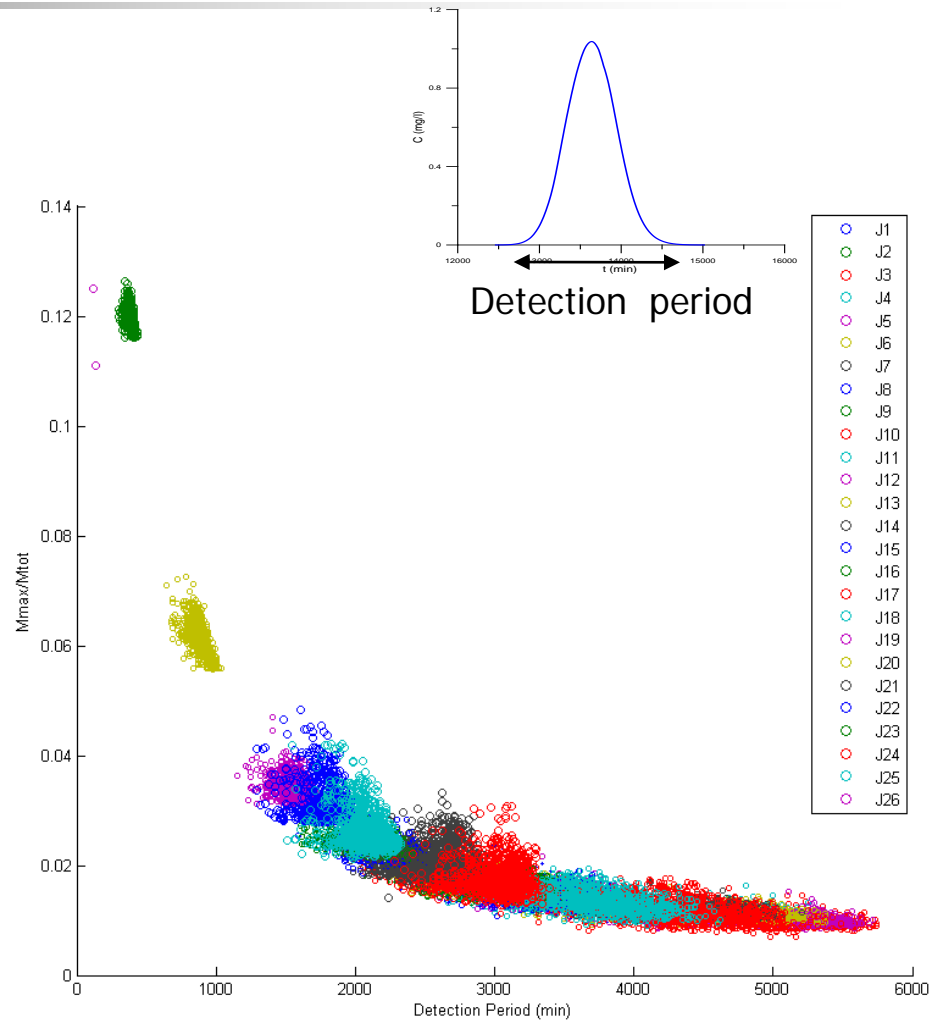
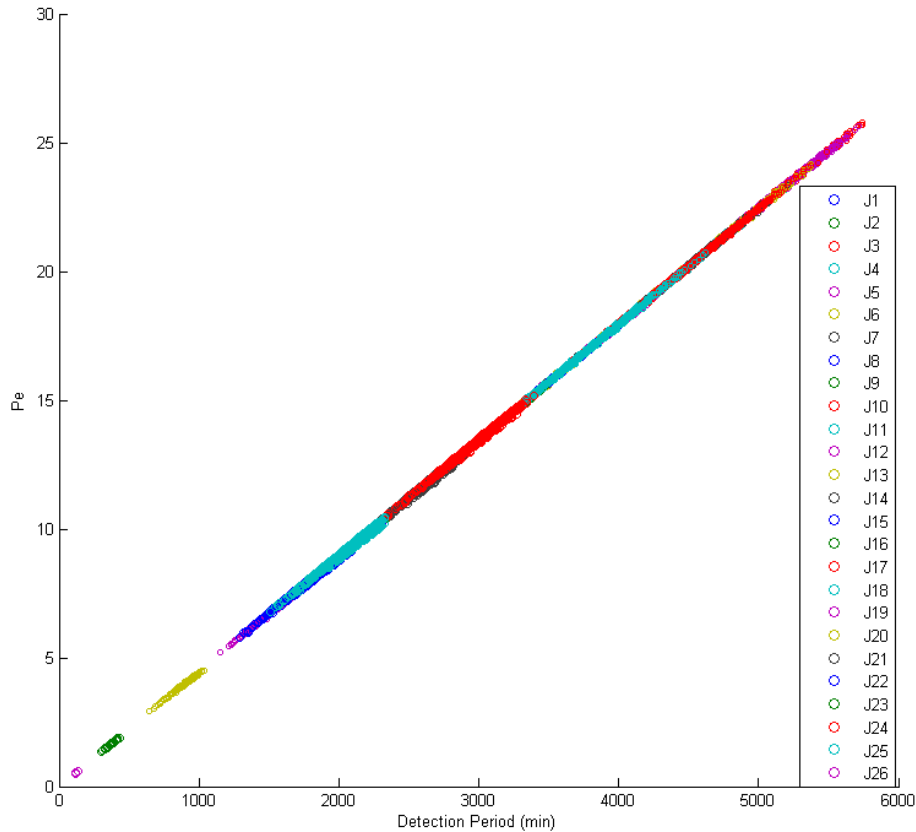
$$\frac{B^2}{dU_*} \propto \frac{L^2}{L^{3/2}} \quad \text{assuming } g \text{ and } S_o \text{ as constants.}$$

Therefore for small changes in depth,

$$Pe \propto \sum_{k:C_k > \text{threshold}} \Delta t = \text{Detection period}$$



Comparison of the two parameters



Pe gives information about support base (or detection period)

M_{max}/M_{tot} gives information about vertical scale of the profile



Scenario Generation



S
ll location.

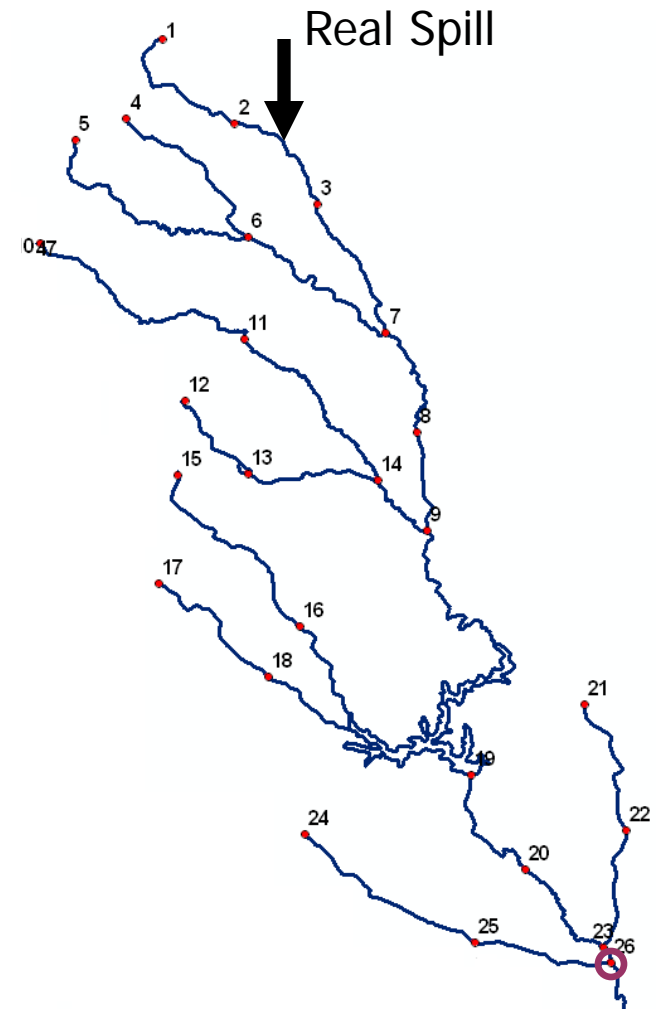
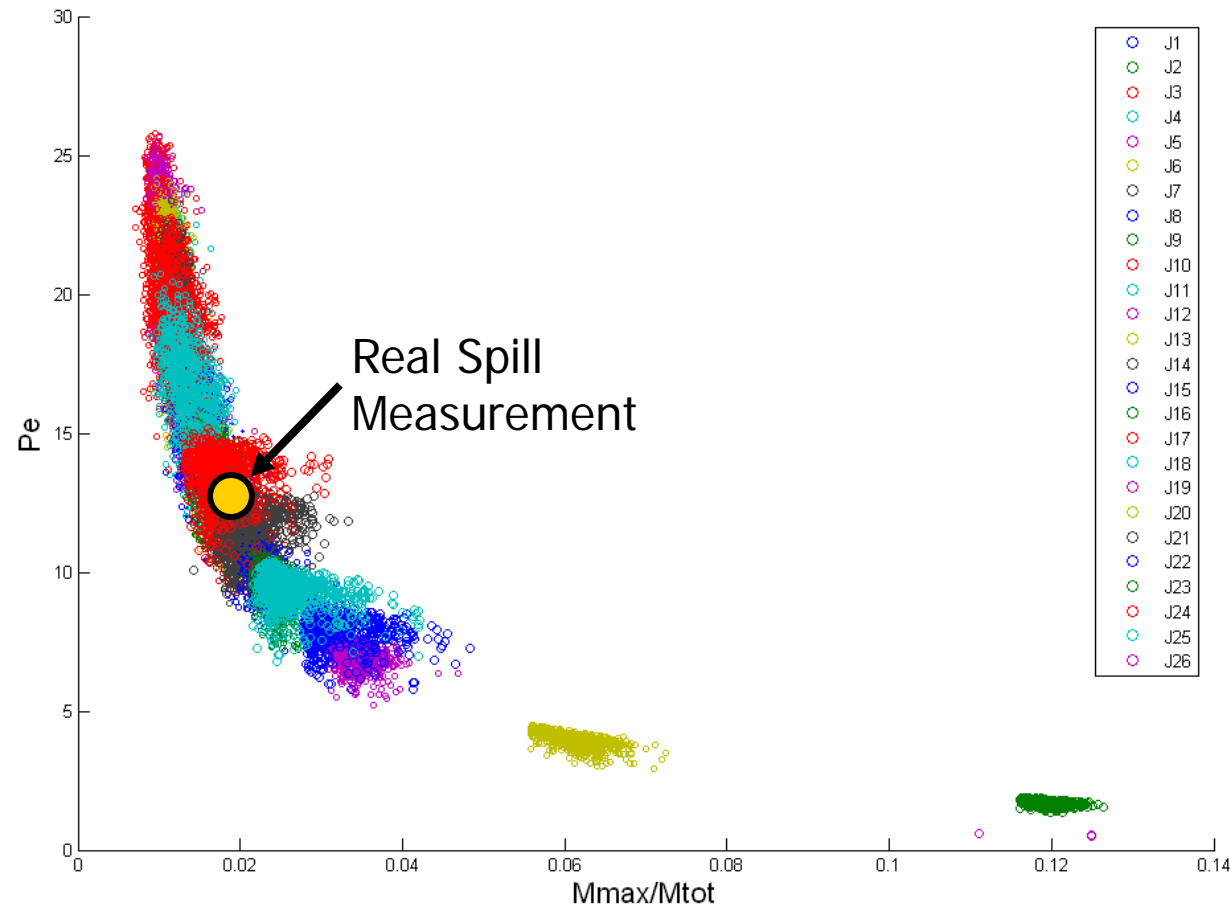
guration
d spill time
e for all

10 days.



Characterization of Scenario Results in terms of Parameter 1 and 2

Region 1 scenarios observed at sensor 1



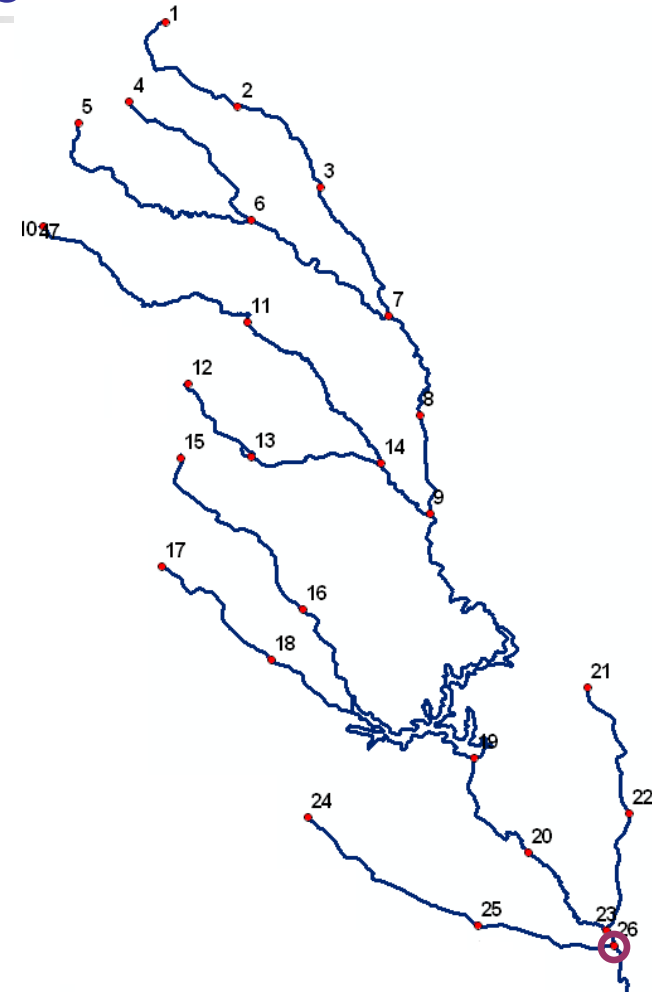
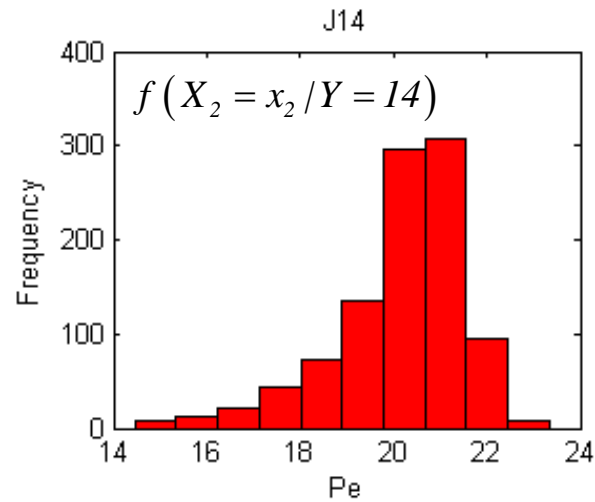
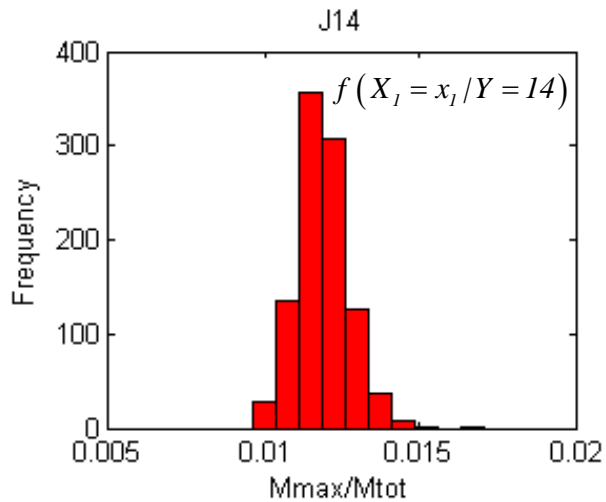
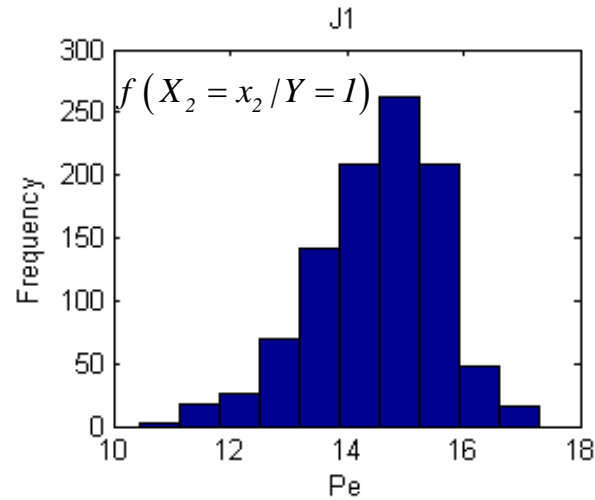
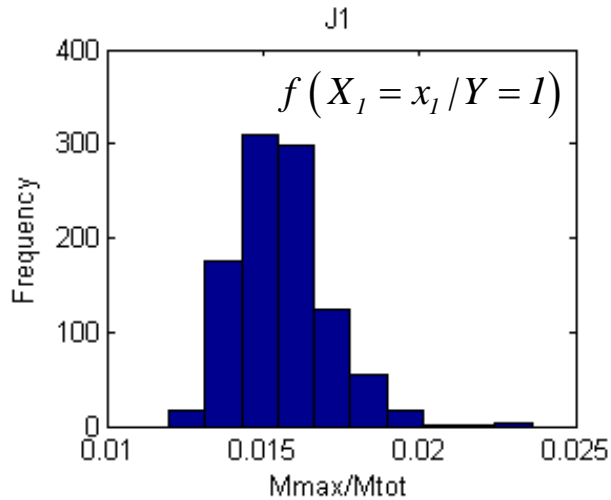
Random Variables

In our analysis a spill must have three main properties:

- $X_1: M_{\max} / M_{\text{tot}}$ observed at the monitoring station. }
 $X_2: P_e$ observed at the monitoring station. } Features of the spill event
- $Y: \text{Junction}$ where the spill has occurred. } Class of the spill event



Conditional prior probability densities



$f(X/Y)$: Conditional probability density function of $X \in \{X_1, X_2\}$ given that spill occurred at Y approximated from Kernel density estimation.

Conditional posterior probabilities

Bayes' Theorem:

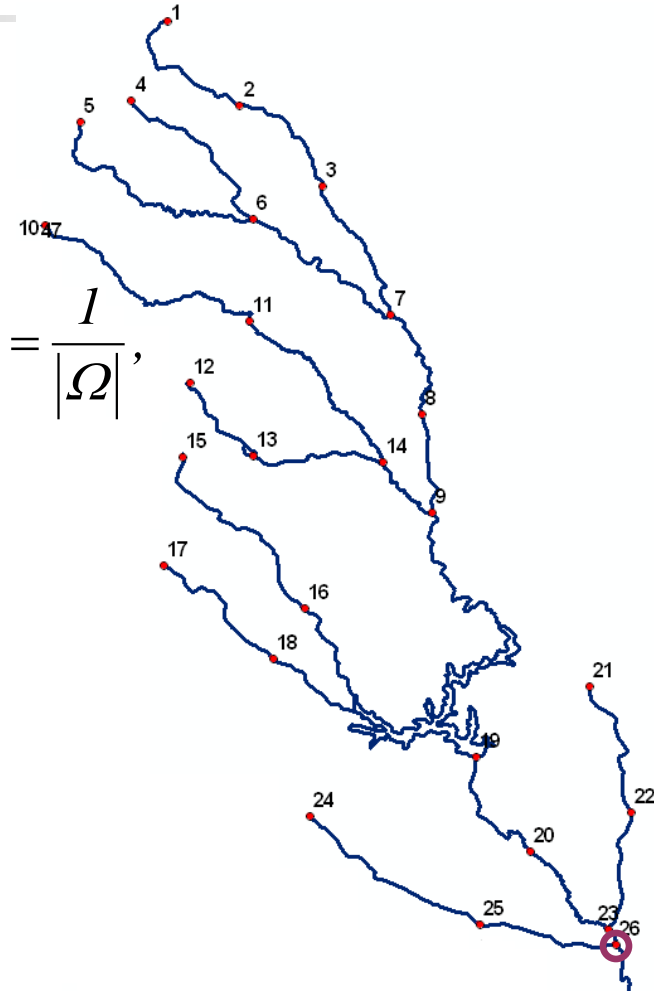
Let $\Omega = \{1, 2, \dots, M\}$ is the set of all classes (junctions)

possible for a spill event. Then, for $j \in \Omega$ and $P(Y = j) = \frac{1}{|\Omega|}$,

$$P(Y = j / X_1 = x_1) = \frac{f(X_1 = x_1 / Y = j)}{\sum_{i \in \Omega} f(X_1 = x_1 / Y = i)}$$

$$P(Y = j / X_2 = x_2) = \frac{f(X_2 = x_2 / Y = j)}{\sum_{i \in \Omega} f(X_2 = x_2 / Y = i)}$$

$P(Y/X)$: Conditional probability of $Y \in \Omega$ given that spill has feature of $X \in \{X_1, X_2\}$



Conditional prior joint probability densities

Copula Concept:

$$f(X_1, X_2 / Y) = c(F(X_1 / Y), F(X_2 / Y)) f(X_1 / Y) f(X_2 / Y)$$

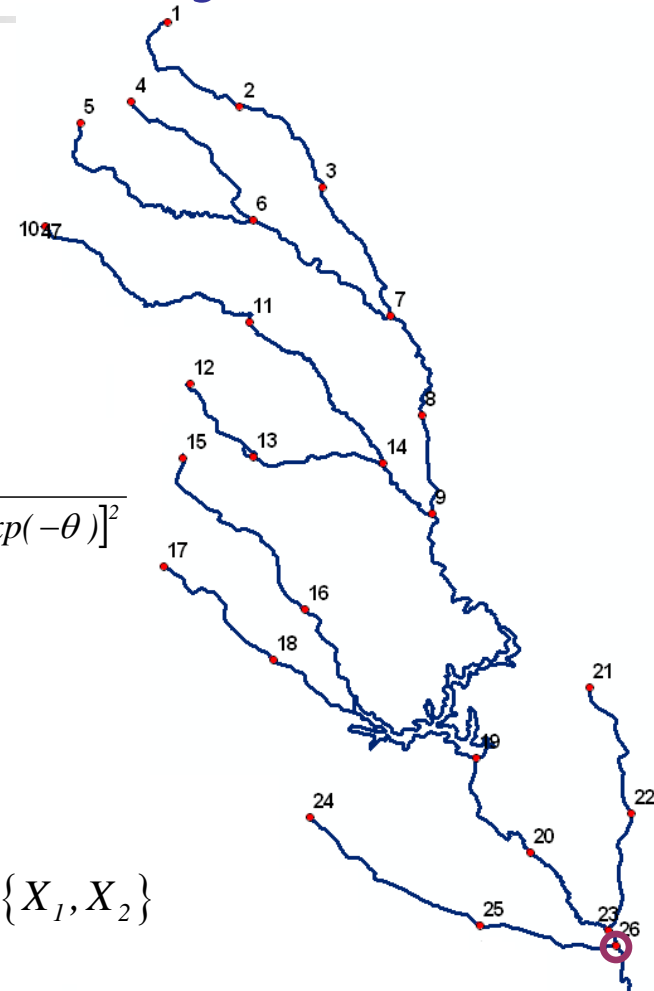
Frank Copula:

$$c(F_X(x), F_Y(y)) = - \frac{\theta \exp(-\theta(F_X(x) + F_Y(y)))(\exp(-\theta) - 1)}{[\exp(-\theta(F_X(x) + F_Y(y))) - \exp(-\theta F_X(x)) - \exp(-\theta F_Y(y)) + \exp(-\theta)]^2}$$

$c(u, v)$: Copula density function, θ : copula parameter

$F(X / Y)$: Conditional marginal cumulative distribution of $X \in \{X_1, X_2\}$ given that spill has occurred at Y.

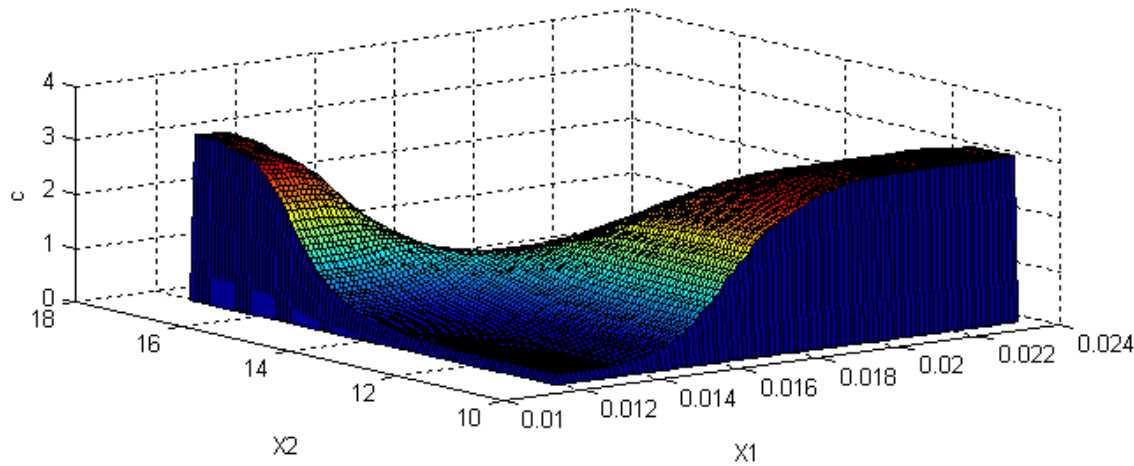
$f(X_1, X_2 / Y)$: Conditional joint probability density of X_1 and X_2 given that spill has occurred at Y.



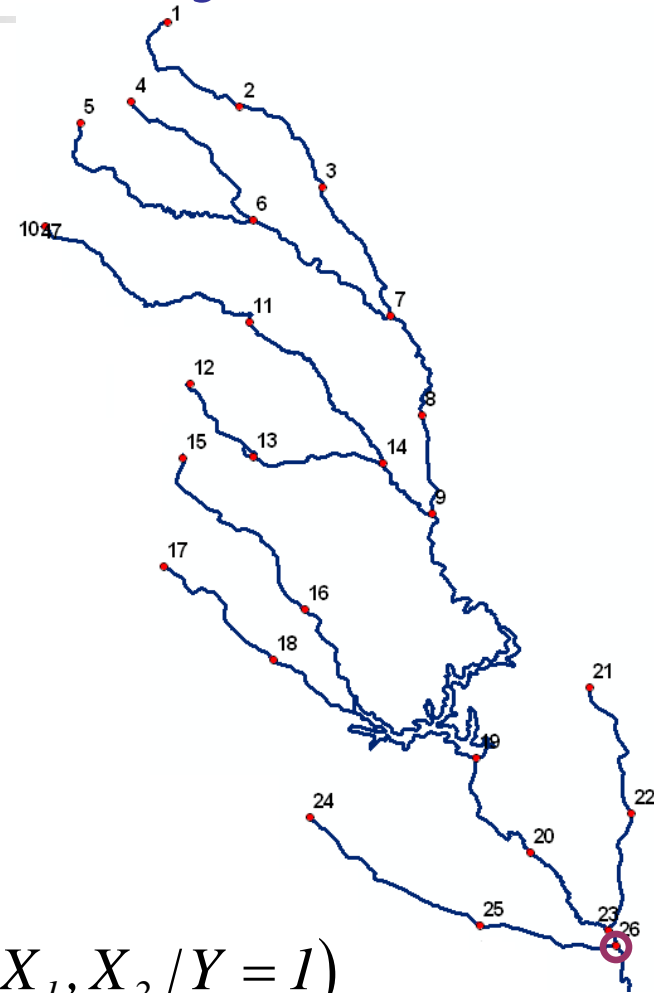
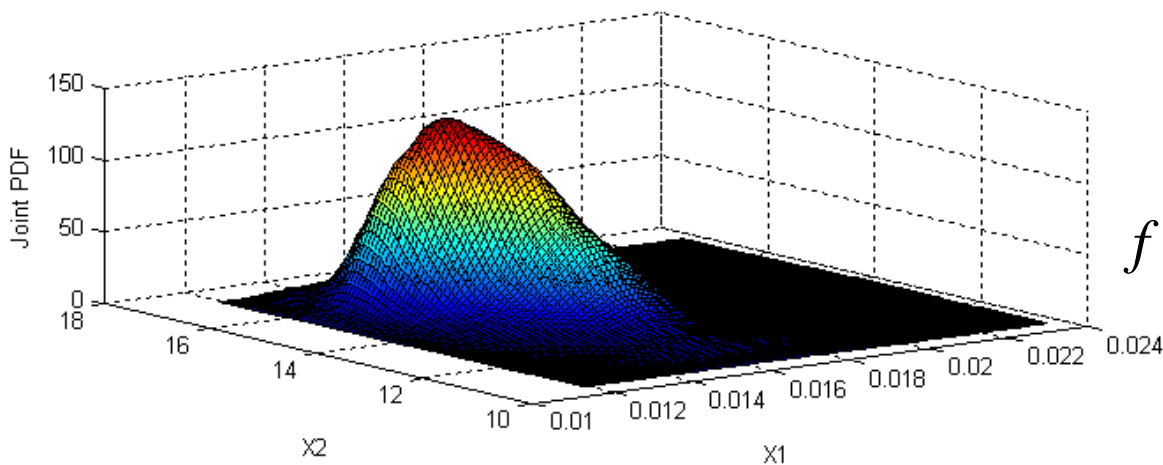
Conditional prior joint probability densities

For spills coming from Junction 1

Copula Density (c)



Joint Probability Distribution



$$f(X_1, X_2 / Y = 1)$$



Conditional posterior joint probabilities

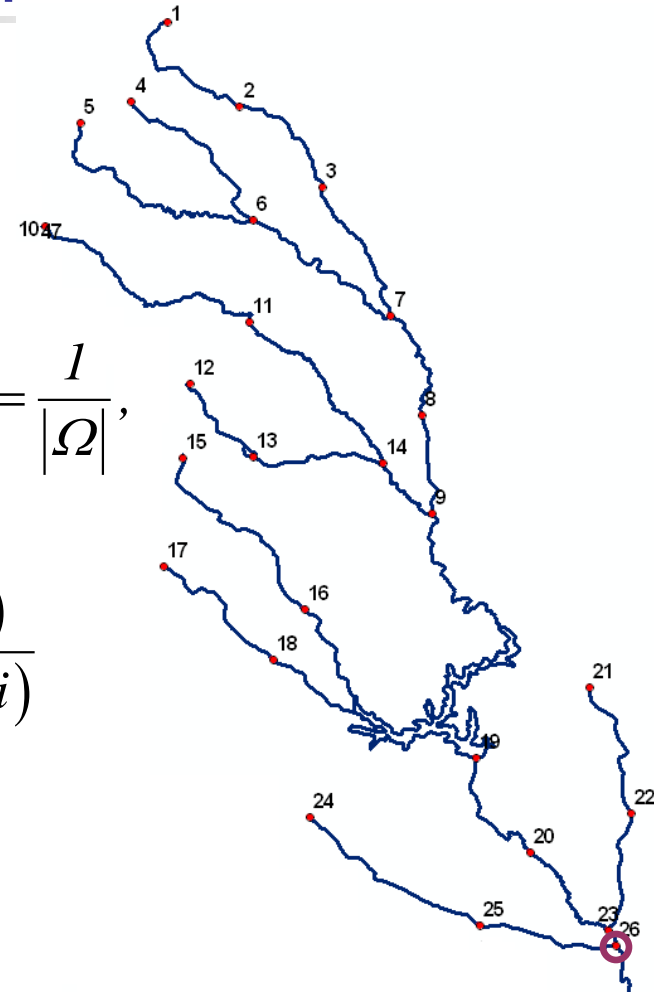
Bayes' Theorem:

Let $\Omega = \{1, 2, \dots, M\}$ is the set of all classes (junctions)

possible for a spill event. Then, for $j \in \Omega$ and $P(Y = j) = \frac{1}{|\Omega|}$,

$$P(Y = j / X_1 = x_1, X_2 = x_2) = \frac{f(X_1 = x_1, X_2 = x_2 / Y = j)}{\sum_{i \in \Omega} f(X_1 = x_1, X_2 = x_2 / Y = i)}$$

$P(Y / X_1, X_2)$: Conditional joint probability of $Y \in \Omega$
given that spill has features of X_1 and X_2 .



Summary of statistical analysis

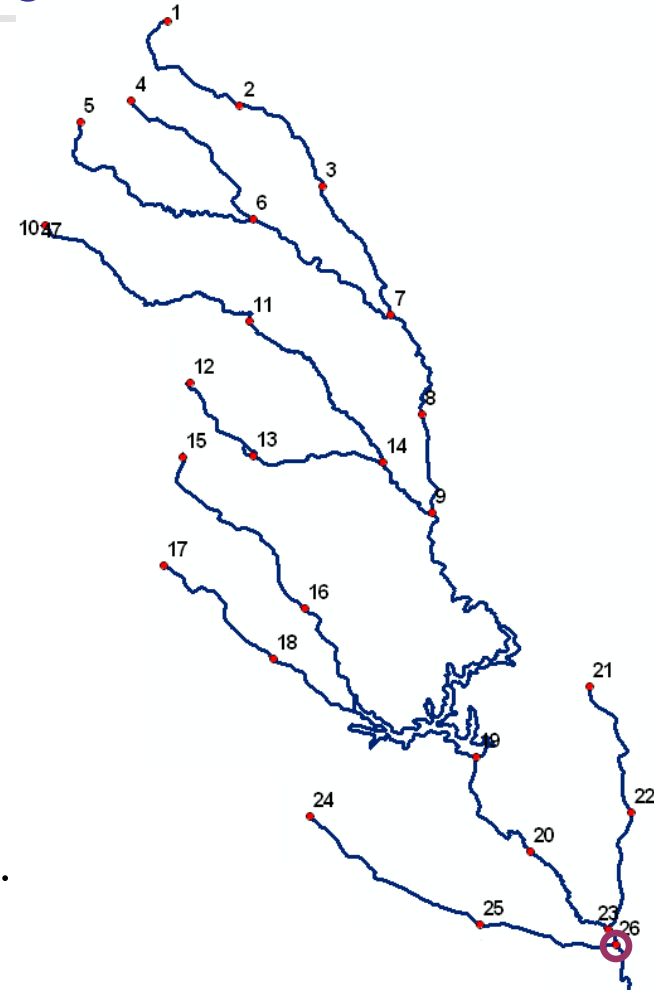
Up to now, we can estimate three conditional probability values:

$P(Y / X_1)$: Conditional probability of $Y \in \Omega$
given that spill has feature of X_1

$P(Y / X_2)$: Conditional probability of $Y \in \Omega$
given that spill has feature of X_2

$P(Y / X_1, X_2)$: Conditional joint probability of $Y \in \Omega$
given that spill has features of X_1 and X_2 .

How can we use this information to determine most possible classes, Y
(or junctions) for a spill event having features x_1 and x_2 ?



Adaptive Sequential Feature Selection

Adaptive Feature Selection in Pattern Recognition and Ultra-Wideband Radar Signal Analysis; Phd Thesis by Hao Jiang ,CalTech, 2008.

Problem Statement:

Let (X, Y) be a pair of random variables: $X \in R^d$, $Y \in \{1, 2, \dots, M\}$

where M is the number of classes or patterns.

$D_n = \{(X^1, Y^1), (X^2, Y^2), \dots, (X^n, Y^n)\}$ is the training set with elements

being i.i.d. samples from a fixed but unknown distribution governing (X, Y) .

$$X = [X_1 \quad X_2 \quad \dots \quad X_d]^T$$

Assume that the classes Y take a prior distribution $\{P(Y = j), j = 1, 2, \dots, M\}$

The goal is to find a mapping $g: R^d \rightarrow \{1, 2, \dots, M\}$ such that an arbitrary

unlabeled test data $x = [x_1 \quad x_2 \quad \dots \quad x_d]^T \in R^d$ can be classified into

one of the M classes while optimizing some criterion.



Adaptive Sequential Feature Selection

Optimum Feature Selection:

Entropy corresponding to prior p.m.f:

$$H_0 = -\sum_{i=1}^M P(Y = i) \log [P(Y = i)], \quad \text{to be unbiased } P(Y = j) = \frac{1}{M}, \quad j = 1, \dots, M$$

Class entropy conditioned on the sample x_i :

$$H_i = -\sum_{j=1}^M P(Y = j / X_i = x_i) \log [P(Y = j / X_i = x_i)]$$

The quantity $H_0 - H_i$ measures how well the feature x_i (which is the i^{th} component of the unlabeled test data x) reduces the complexity of the classification task.

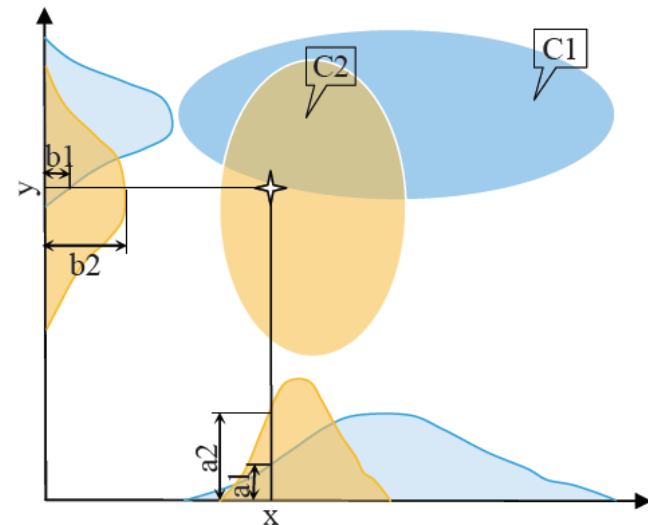
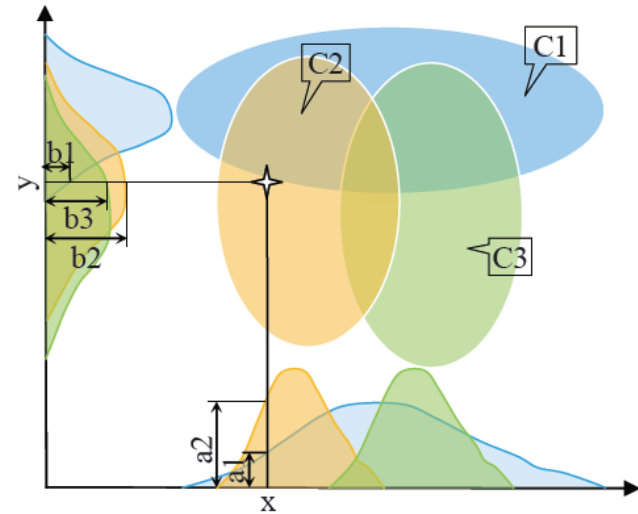
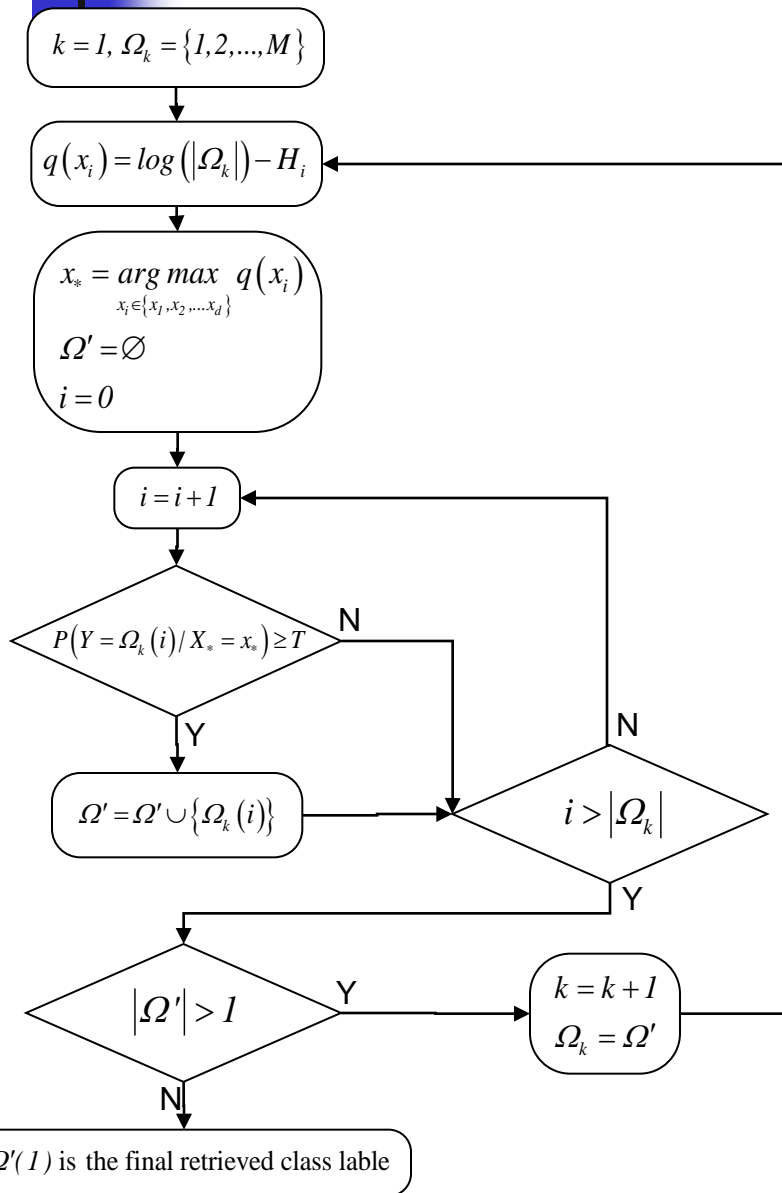
$$q(x_i) = H_0 - H_i$$

The best feature in terms of reducing the class entropy can be selected as:

$$x_* = \operatorname{argmax}_{x_i \in \{x_1, \dots, x_d\}} q(x_i)$$



Adaptive Sequential Feature Selection



Jiang (2008)





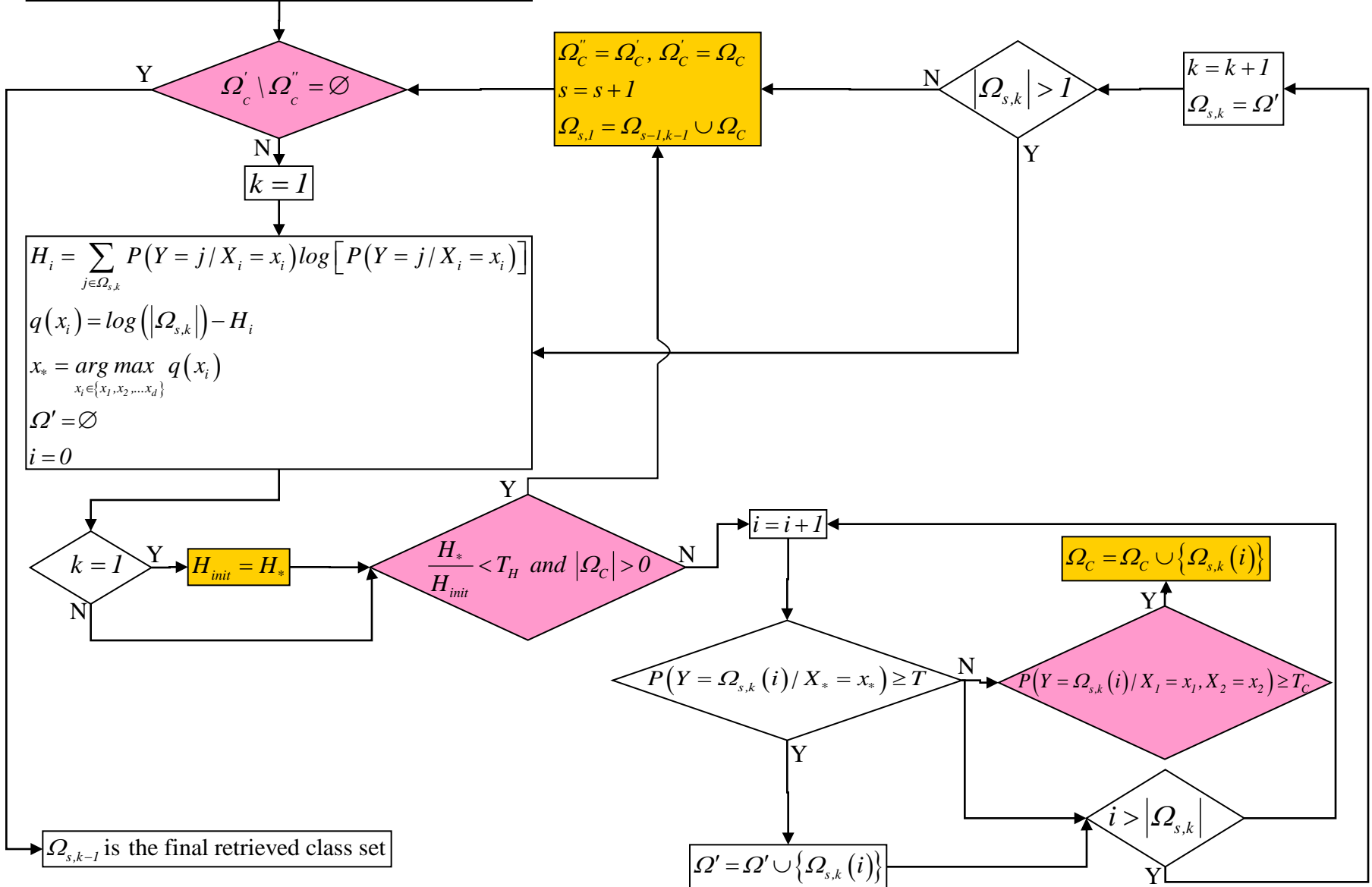
Adaptive Sequential Feature Selection

- Original form of ASFS algorithm reduces the initial class set to one final selected class by eliminating the classes using a probability threshold.
- However, final result is not always the correct answer.
- We have made some modifications to add some flexibility to ASFS algorithm.
 - We introduced a copula set Ω_C in which we store some eliminated classes according to their performance against a joint probability threshold.
 - We determined a criterion to stop ASFS algorithm at some level where the correct class is in the final set.
 - At the end of one ASFS cycle (according to our new stopping criteria), we added copula set into final class set obtained and re-ran ASFS algorithm.
 - This modified ASFS algorithm is stopped when two consecutive copula sets obtained are the same.

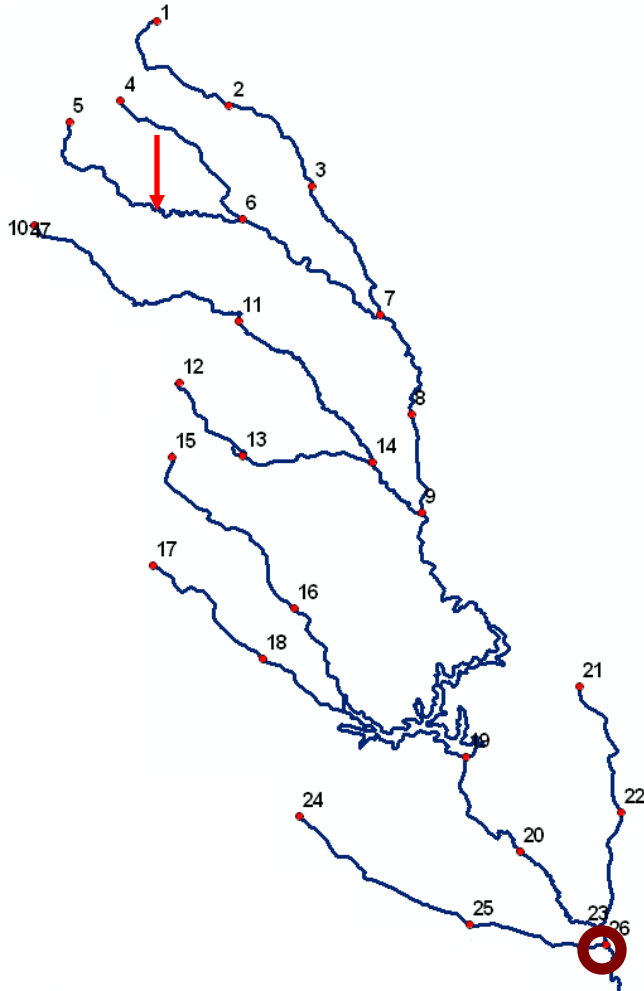


Modified ASFS

$$\Omega_{1,1} = \{1, 2, \dots, M\}, \Omega_c = \emptyset, \Omega'_c = \{a\}, \Omega''_c = \{b\}, s = 1$$



Results

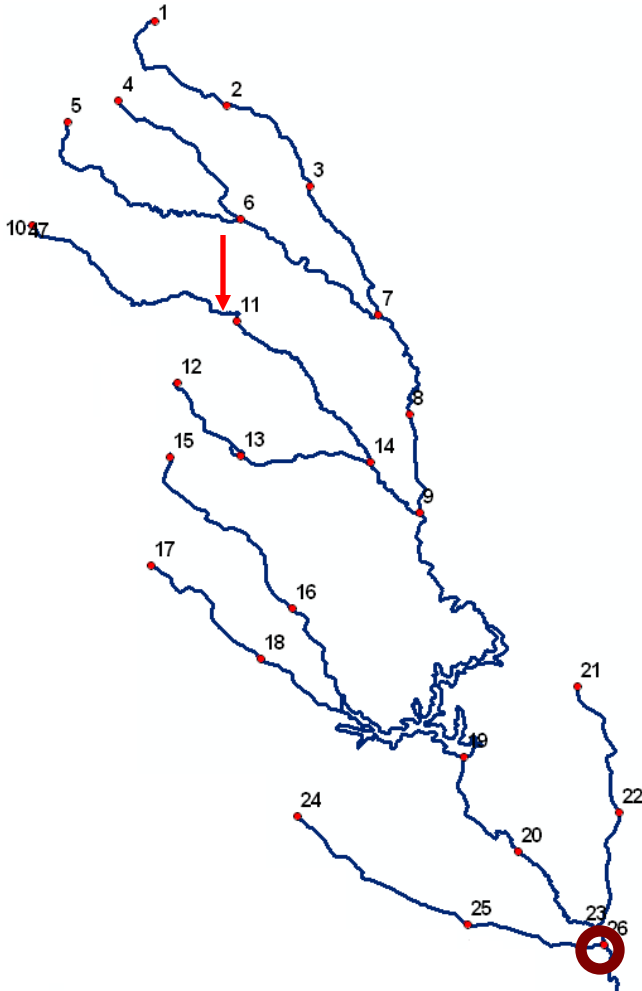


Spill_number:	2																								
Spill_between:	5					6																			
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	P	M	M	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.98	0.89	0.86	0.76
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	6
	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	6	6	6	6	7
	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6	6	6	7
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7	21
	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8	8	8	8	24
	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	9	9	9	9	24
	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	15	16	16	16	24
	10	10	11	11	11	11	11	11	11	11	11	11	11	14	15	16	17	18	18	18	21	21	24	24	24
	11	11	12	12	13	13	13	13	14	15	16	17	18	21	21	24									
	12	12	13	13	14	14	14	14	15	16	17	18	21	21	24										
	13	13	14	14	15	15	15	16	17	18	21	21	24	25											
	14	14	15	15	16	16	16	16	17	18	21	24	25												
	15	15	16	16	17	17	17	17	18	21	24	25													
	16	16	17	17	18	18	18	18	21	24	25														
	17	17	18	18	19	19	19	19	21	24	25														
	18	18	19	19	21	21	21	24	25																
	19	19	21	21	22	24	25																		
	20	21	22	22	24	25																			
	21	22	24	24	25																				
	22	24	25	25																					
	23	25	26																						
	24	26																							
	25																								
	26																								

Spill_number:	2	
Spill_between:	5	6
FS	M	M
Hr	1	1
	1	1
	2	2
	3	3
	4	4
	5	6
	6	7
	7	16
	16	21
	21	24
	24	

Result_for_spill_2_wrt_5_and_6	J	C	MM	Pe	Avg
	1	0.04	0.01	0.13	0.06
	2	0.2	0.07	0.25	0.18
	3	0.09	0.19	0.1	0.13
	6	0.12	0.03	0.2	0.11
	7	0	0.43	0.01	0.15
	21	0	0.18	0	0.06
	24	0.55	0.1	0.32	0.32

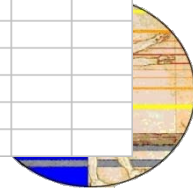
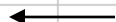
Results



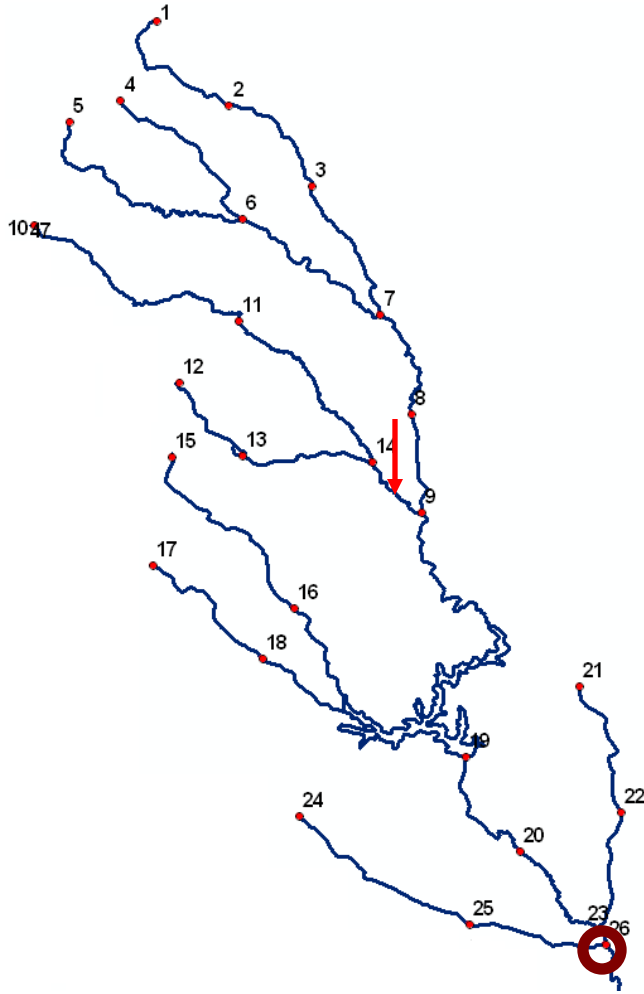
Spill_number:	4																
Spill_between:	10		11														
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	0.91	0.78
	1	1	1	1	1	1	1	1	1	4	4	4	10	10	11	11	
	2	2	2	2	2	2	4	4	5	5	10	11	11	12	13	14	
	3	3	3	4	4	4	5	5	10	11	11	12	12	13	14	17	
	4	4	4	5	5	5	6	10	11	12	12	13	14	14	17		
	5	5	5	6	6	6	10	11	12	12	13	14	17	17			
	6	6	6	10	10	10	11	12	13	13	14	17	17				
	7	7	10	11	11	11	12	13	14	14	17	18					
	8	10	11	12	12	12	13	14	15	17	18						
	9	11	12	13	13	13	14	15	17	18							
	10	12	13	14	14	14	15	17	18								
	11	13	14	15	15	15	17	18									
	12	14	15	16	16	17	18										
	13	15	16	17	17	18											
	14	16	17	18	18												
	15	17	18	24													
	16	18	24														
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	20																
	21																
	22																
	23																
	24																
	25																
	26																

Spill_number:	4		
Spill_between:	10	11	
FS	P	M	P
Hr	1	0.86	0.7
	11	11	11
	12	13	13
	13	14	14
	14	17	
	17		

Result_for_spill_4_wt_10_an_11	J	C	MM	Pe	Avg
	11	0.31	0.35	0.23	0.3
	13	0.32	0.34	0.25	0.3
	14	0.27	0.18	0.37	0.27
	17	0.1	0.13	0.15	0.13



Results



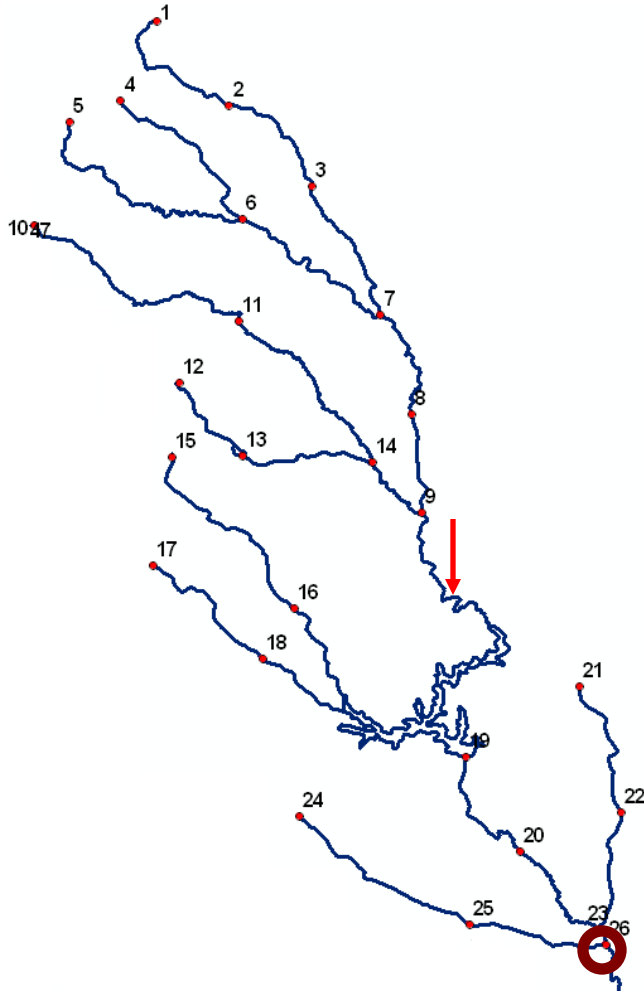
Spill_number:	5															
Spill_between:	14 9															
FS	P	P	P	P	P	P	P	P	P	P	P	M	P	M		
Hr	1	1	1	1	1	1	1	1	1	1	1	0.97	0.93	0.87	0.83	
	1	1	1	1	1	1	1	1	1	4	4	4	4	11	11	
	2	2	2	2	2	2	2	4	4	5	5	10	11	12	13	
	3	3	3	4	4	4	4	5	5	10	10	11	12	13	14	
	4	4	4	5	5	5	6	10	11	11	11	12	13	14	17	
	5	5	5	6	6	6	10	11	12	12	13	14	17	18		
	6	6	6	10	10	10	11	12	13	13	14	17	18			
	7	7	10	11	11	11	12	13	14	14	17	18				
	8	10	11	12	12	12	13	14	15	17	18					
	9	11	12	13	13	13	14	15	17	18						
	10	12	13	14	14	14	15	17	18							
	11	13	14	15	15	15	17	18								
	12	14	15	16	16	17	18									
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	19															
	20															
	21															
	22															
	23															
	24															
	25															
	26															

Spill_number:	5			
Spill_between:	14 9			
FS	M	M	P	P
Hr	1	0.98	0.94	0.81
	10	11	11	11
	11	12	13	13
	12	13	14	14
	13	14	17	17
	14	17	18	
	17	18		
	18			

Result_for_spill	5	wt	14	an	9												
	J	C	MM	Pe	Avg												
	11	0.1	0.09	0.17	0.12												
	13	0.11	0.1	0.16	0.12												
	14	0.43	0.28	0.29	0.33												
	17	0.29	0.19	0.32	0.27												
	18	0.08	0.34	0.06	0.16												



Results

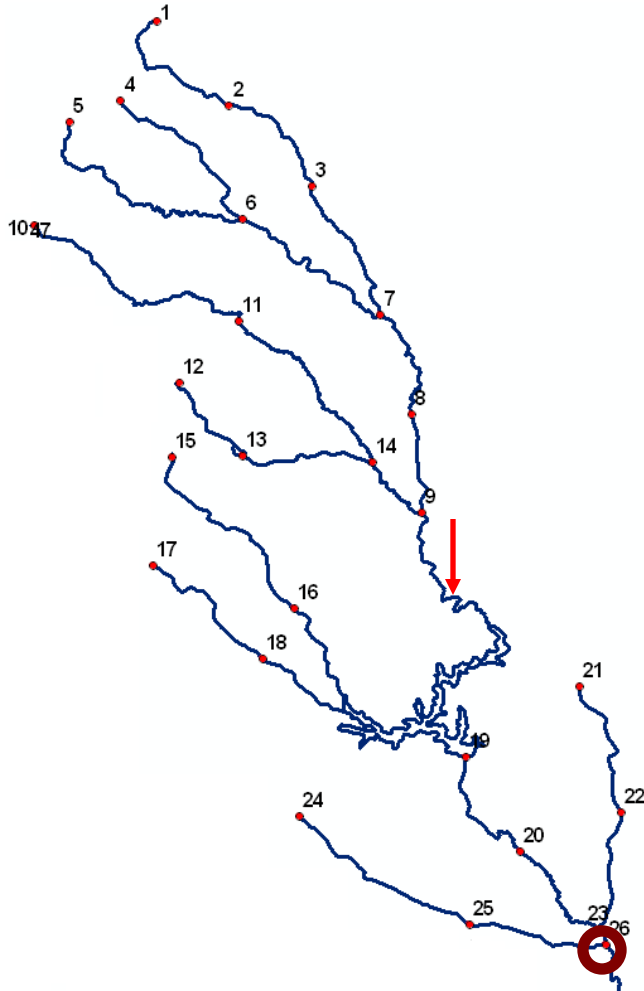


Spill_number:	6																		
Spill_between:	9 19																		
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	P	P	P	P		
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.84	0.84	0.83	0.81	0.73
	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	3	7	7
	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	7	8	8
	3	3	3	3	3	3	3	3	3	3	3	3	3	7	7	7	8	9	9
	4	4	4	4	4	4	5	5	7	7	7	7	7	8	8	8	9	21	25
	5	5	5	5	5	5	6	7	8	8	8	8	8	9	9	9	21	25	
	6	6	6	6	6	6	7	8	9	9	9	9	9	21	21	21	25		
	7	7	7	7	7	7	8	9	17	18	19	21	22	24	25				
	8	8	8	8	8	8	9	17	18	19	21	22	24	25					
	9	9	9	9	9	9	17	18	19	21	22	24	25						
	10	15	15	16	17	18	19	21	22	24	25								
	11	16	16	17	18	19	21	22	24	25									
	12	17	17	18	19	21	22	24	25										
	13	18	18	19	21	22	24	25											
	14	19	19	21	22	24	25												
	15	21	21	22	24	25													
	16	22	22	24	25														
	17	24	24	25															
	18	25	25																
	19	26																	
	20																		
	21																		
	22																		
	23																		
	24																		
	25																		
	26																		

Spill_number:	6			
Spill_between:	9 19			
FS	P	M	M	M
Hr	1	0.9	0.9	0.7
	7	7	8	9
	8	8	9	25
	9	9	25	
	21	25		
	25			

Result_for_spill	6	9	and	19	
	J	C	MM	Pe	Avg
	8	0.02	0.06	0.07	0.05
	9	0.54	0.52	0.39	0.48
	25	0.45	0.42	0.54	0.47

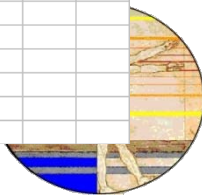
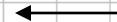
Results



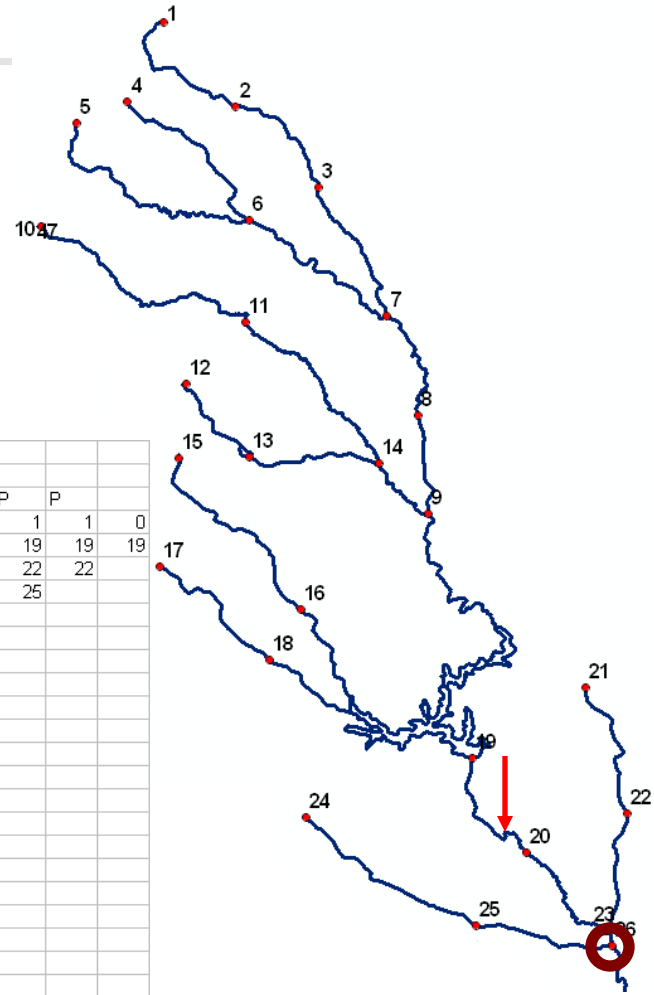
Spill_number:	7																			
Spill_between:	18		19																	
FS	P	P	P	P	P	P	P	P	P	P	M	M	M	M	M	M	M	P		
Hr	1	1	1	1	1	1	1	1	1	1	0.94	0.94	0.94	0.94	0.93	0.91	0.89	0.86	0.83	
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	6
	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	16
	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6	18
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	15	16	18	24
	7	7	7	7	7	7	7	7	7	7	11	11	14	15	15	16	18	24		
	8	8	8	8	8	10	11	11	11	11	13	14	15	16	16	18	24			
	9	9	9	10	10	11	12	12	12	13	14	15	16	17	18	24				
	10	10	10	11	11	12	13	13	13	14	15	16	17	18	24					
	11	11	11	12	12	13	14	14	15	16	17	18	24							
	12	12	12	13	13	14	15	15	16	17	18	24								
	13	13	13	14	14	15	16	16	17	18	24									
	14	14	14	15	15	16	17	17	18	24										
	15	15	15	16	16	17	18	18	24											
	16	16	16	17	17	18	21	24												
	17	17	17	18	18	21	24													
	18	18	18	21	21	24														
	19	21	21	24	24															
	20	22	24	25																
	21	24	25																	
	22	25																		
	23																			
	24																			
	25																			
	26																			

Spill_number:	7			
Spill_between:	18		19	
FS	M	P	M	P
Hr	1	0.97	0.93	0.86
	1	1	1	1
	2	2	2	2
	3	3	6	6
	5	6	16	16
	6	16	18	24
	16	18	24	
	18	24		
	24			

Result_for_spill	7_wf	18_an	19		
	J	C	MM	Pe	Avg
	1	0.1	0.1	0.14	0.11
	2	0.18	0.34	0.16	0.23
	6	0.16	0.16	0.2	0.17
	16	0.21	0.15	0.22	0.19
	18	0.04	0.06	0.03	0.04
	24	0.31	0.19	0.26	0.25



Results



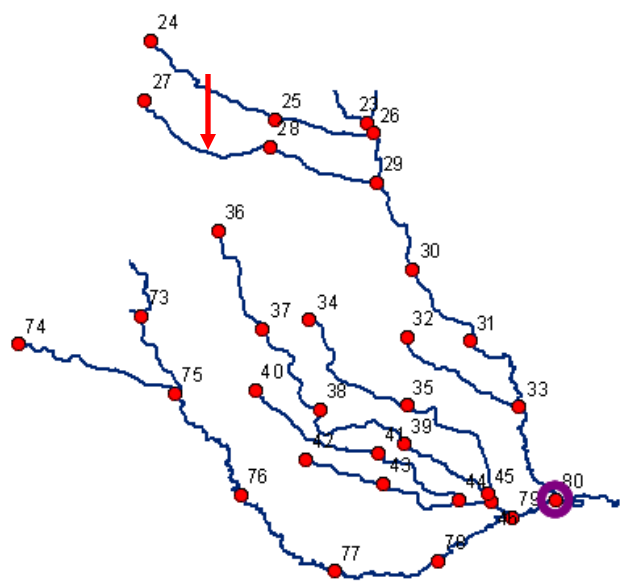
Spill_number:	8																								
Spill_between:	19																			20					
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	19
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	19
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	19
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	19
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	19
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	19
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	19
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	19
10	10	10	10	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	19
11	11	11	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	19
12	12	12	12	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	19
13	13	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	19
14	14	14	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	19
15	15	15	15	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	19
16	16	16	16	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	19
17	17	17	17	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	19
18	18	18	18	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19
19	19	19	19	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	19
20	20	20	20	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	19
21	21	21	21	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	19
22	22	22	22	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	19
23	23	23	23	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	19
24	24	24	24	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	19
25	25	25	25	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	19
26	26	26	26	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	19

Result_for_spill	8	19	an	20	
	J	C	MM	Pe	Avg
19	0.57	0.15	0.88	0.53	
22	0.43	0.85	0.12	0.47	



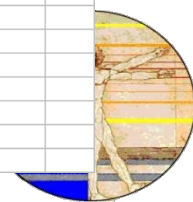
Results

Spill_number	1																			
Spill_between	27	28																		
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Hr	1	1	1	1	1	1	1	1	1	1	0.99	0.98	0.97	0.95	0.94	0.89	0.85			
27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27			
28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28			
29	29	29	29	29	32	32	32	34	34	34	34	34	34	34	35	35	35			
30	30	32	32	34	34	34	34	35	35	35	35	35	35	35	37	38	39			
31	32	34	34	35	35	35	36	37	37	37	37	37	37	37	38	39	40			
32	34	35	35	36	36	36	37	37	37	38	38	38	38	38	39	40	43			
33	35	36	36	37	37	37	38	39	39	39	39	39	39	40	43	45				
34	36	37	37	38	38	38	39	40	40	40	40	40	40	40	43	45				
35	37	38	38	39	39	39	39	40	41	41	41	41	41	43	45					
36	38	39	39	40	40	40	40	41	42	42	43	43	45							
37	39	40	40	41	41	41	41	42	43	43	45									
38	40	41	41	42	42	42	43	45	45											
39	41	42	42	43	43	43	45	74												
40	42	43	43	44	44	44	45	74												
41	43	44	44	45	45	74														
42	44	45	45	46	74															
43	45	46	46	74																
44	46	74	74																	
45	74	75																		
46	75																			
74																				
75																				
76																				
77																				
78																				
79																				
80																				

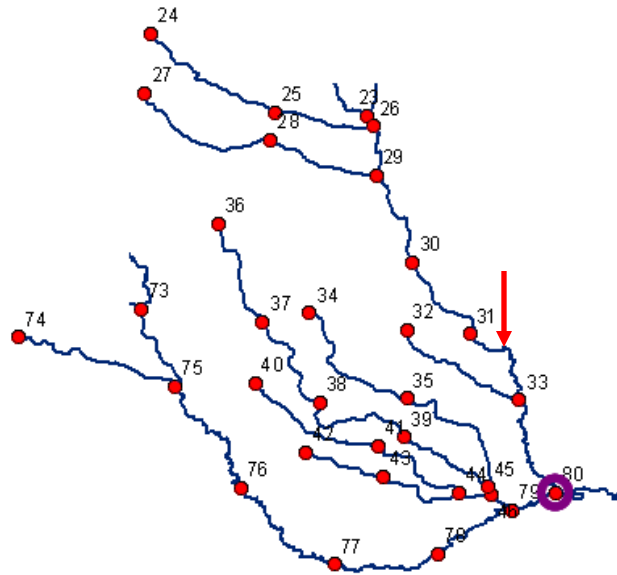


Spill_number	1				
Spill_between	27	28			
FS	P	P	P	P	P
Hr	1	0.99	0.97	0.92	0.85
27	27	27	27	27	27
28	28	28	28	35	39
35	35	35	35	39	40
39	39	39	39	40	43
40	40	40	40	43	45
41	41	43	45		
42	43	45			
43	45				
45					

Result	for	spill	numbe	1	which	is	betwe	27	and	28
	J	C	MM	Pe	Avg					
	27	0.065	0.09	0.077	0.077	←				
	35	0.028	0.05	0.046	0.043					
	39	0.231	0.28	0.17	0.227					
	40	0.095	0.15	0.11	0.118					
	43	0.203	0.15	0.241	0.199					
	45	0.378	0.28	0.355	0.337					

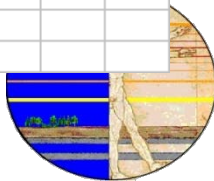


Results

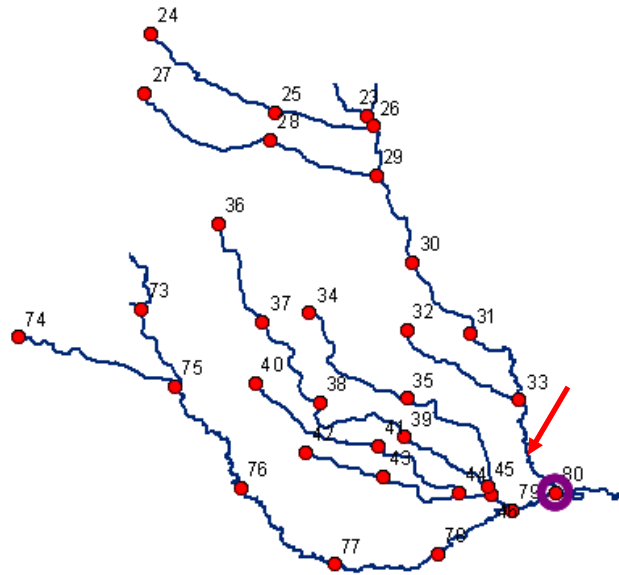


Spill_number	3																	
Spill_between	31 33																	
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27	29	29	29	29	30	30	30	30	30	30	30	30	30	30	30	30	30	30
28	30	30	30	30	31	31	31	31	31	31	31	31	31	31	31	31	31	31
29	31	31	31	31	33	33	33	33	33	33	33	33	33	33	33	33	33	33
30	33	33	33	33	41	43	43	43	43	44	46	46	46	46	46	46	46	46
31	40	41	41	41	43	44	44	44	44	46	46	46	46	46	46	46	46	46
32	41	42	43	43	44	46	46	46	46	46	46	46	46	46	46	46	46	46
33	42	43	44	44	46	46	46	46	46	46	46	46	46	46	46	46	46	46
34	43	44	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
35	44	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
36	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
37	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
38	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
39	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
40	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
41	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
42	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
43	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
44	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
45	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
74	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
75	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
76	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
77	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
78	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
79	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
80	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46

Result	for	spill	numbe	3	which	is	betwe	31	and	33
	J	C	MM	Pe	Avg					
	31	0.745	0.69	0.683	0.705	←				
	77	0.255	0.31	0.317	0.295					

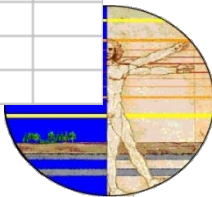


Results

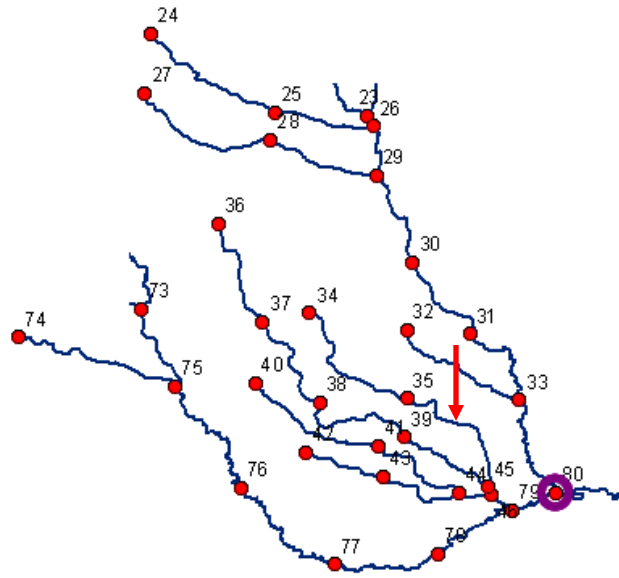


Spill_number	5				
Spill_between	33	80			
FS	M	M	M	P	
Hr	1	1	1	0	0
	27	33	33	79	79
	28	78	79	80	
	29	79	80		
	30	80			
	31				
	32				
	33				
	34				
	35				
	36				
	37				
	38				
	39				
	40				
	41				
	42				
	43				
	44				
	45				
	46				
	74				
	75				
	76				
	77				
	78				
	79				
	80				

Result	for	spill	numbe	5	which	is	betwe	33	and	80
	J	C	MM	Pe	Avg					
	79	0	0	1	0.5					
	80	0	1	0	0.5		←			



Results



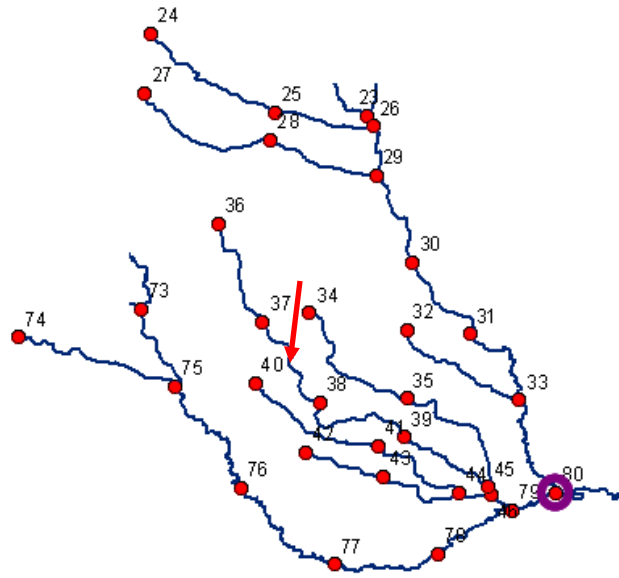
Spill_number	6																			
Spill_between	35	45																		
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.98	0.96	0.89	0.81		
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	34	34	34	34		
	28	28	28	28	34	34	34	34	34	34	34	34	34	34	35	35	35	35		
	29	32	34	34	35	35	35	35	35	35	35	35	35	35	36	36	36	36		
	30	34	35	35	36	36	36	36	36	36	36	36	36	36	37	37	37	37		
	31	35	36	36	37	37	37	37	37	37	37	37	37	37	38	38	38	38		
	32	36	37	37	38	38	38	38	38	38	38	38	38	38	42	42				
	33	37	38	38	39	39	39	39	39	40	42	42	74							
	34	38	39	39	40	40	40	40	42	43	74									
	35	39	40	40	41	42	42	43	74											
	36	40	41	41	42	43	43	74												
	37	41	42	42	43	45	74													
	38	42	43	43	45	74														
	39	43	44	45	74															
	40	44	45	74																
	41	45	74																	
	42	74																		
	43																			
	44																			
	45																			
	46																			
	74																			
	75																			
	76																			
	77																			
	78																			
	79																			
	80																			

Spill_number	6																			
Spill_between	35	45																		
FS	P	P	P																	
Hr	1	0.93	0.84																	
	34	34	34																	
	35	35	35																	
	36	36	36																	
	37	37	37																	
	38	38	38																	
	42	42																		
	74																			

Result	for	spill	numbe	6	which	is	betwe	35	and	45										
	J	C	MM	Pe	Avg															
		34	0.132	0.17	0.134	0.147														
		35	0.219	0.15	0.168	0.179	←													
		36	0.066	0.14	0.082	0.095														
		37	0.244	0.23	0.254	0.243														
		38	0.303	0.19	0.28	0.259														
		42	0.037	0.12	0.082	0.078														



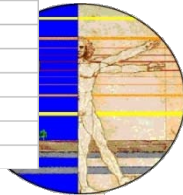
Results



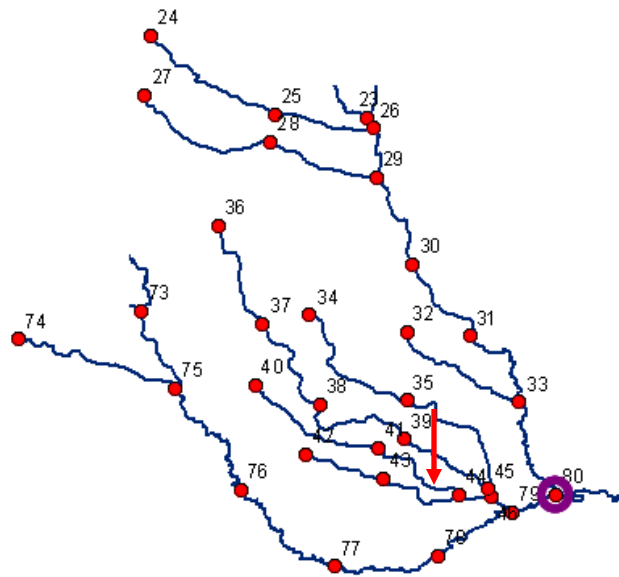
Spill_number	7													
Spill_between	37	38												
FS	P	P	P	P	P	P	P	M	P	M				
Hr	1	1	1	1	1	1	1	0.99	0.93	0.89	0.83			
	27	27	27	27	27	27	27	27	27	27	27			
	28	28	28	34	34	34	34	34	34	34	34			
	29	34	34	35	35	35	35	35	35	35	35			
	30	35	35	36	36	36	36	36	36	36	36			
	31	36	36	37	37	37	37	37	37	37	37			
	32	37	37	38	38	38	38	38	38	38	38			
	33	38	38	39	39	39	39	40	42	42	42			
	34	39	39	40	40	40	40	42	43	43				
	35	40	40	41	42	42	43	74						
	36	41	41	42	43	43	74							
	37	42	42	43	45	74								
	38	43	43	45	74									
	39	44	45	74										
	40	45	74											
	41	74												
	42													
	43													
	44													
	45													
	46													
	74													
	75													
	76													
	77													
	78													
	79													
	80													

Spill_number	7													
Spill_between	37	38												
FS	M	M	P	P										
Hr	1	0.98	0.91	0.83										
	27	27	27	34										
	34	34	34	35										
	35	35	35	37										
	36	36	37	38										
	37	37	38	42										
	38	38	42											
	42	42												
	74													

Result	for	spill	numbe	7	which	is	betwe	37	and	38				
	J	C	MM	Pe	Avg									
	27	0.018	0.27	0.009	0.099									
	34	0.09	0.05	0.159	0.098									
	35	0.22	0.27	0.162	0.217									
	37	0.255	0.1	0.292	0.216									
	38	0.339	0.23	0.281	0.283									
	42	0.078	0.08	0.098	0.086									



Results

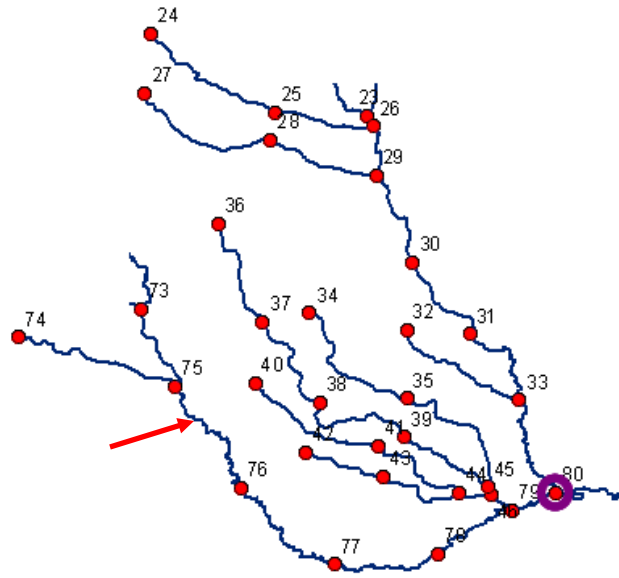


Spill_number	8																						
Spill_between	43										44												
FS	P	P	P	P	P	P	P	M	M	M	M	M	M	M	M	M	M	P	M	M	M		
Hr	1	1	1	1	1	1	1	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.83	0.79	0.74	0.67	0.52	
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	28	28	28	28	28	
	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	32	32	32	32	32	41
	29	29	29	29	29	29	29	32	32	32	32	32	32	32	32	32	32	40	40	40	40	41	43
	30	30	30	32	32	32	32	34	34	35	35	35	37	38	39	40	41	41	41	41	41	43	
	31	32	32	34	34	34	34	35	35	36	37	38	39	40	41	42	42	42	42	42	43		
	32	34	34	35	35	35	35	36	36	37	38	39	40	41	42	43	43	43	43	43			
	33	35	35	36	36	36	36	37	37	38	39	40	41	42	43	44	44						
	34	36	36	37	37	37	37	38	38	39	40	41	42	43	44	45							
	35	37	37	38	38	38	38	39	39	40	41	42	43	44	45								
	36	38	38	39	39	39	39	40	40	41	42	43	44	45									
	37	39	39	40	40	40	40	41	41	42	43	44	45										
	38	40	40	41	41	41	41	42	42	43	44	45											
	39	41	41	42	42	42	42	43	43	44	45												
	40	42	42	43	43	43	43	44	44	45													
	41	43	43	44	44	44	44	45	45														
	42	44	44	45	45	45	45	74															
	43	45	45	46	46	46	74																
	44	46	46	74	74																		
	45	74	74	75																			
	46	75	75																				
	74	76																					
	75																						
	76																						
	77																						
	78																						
	79																						
	80																						

Spill_number	8	
Spill_between	43	44
FS	M	P
Hr	1	0.76
	28	28
	32	41
	41	43
	43	

Result	for	spill	numbe	8	which	is	betwe	43	and	44
	J	C	MM	Pe	Avg					
	28	0.129	0.11	0.27	0.169					
	32	0.191	0.1	0.412	0.233					
	41	0.565	0.65	0.266	0.493					
	43	0.115	0.15	0.053	0.105	←				

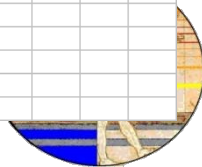
Results



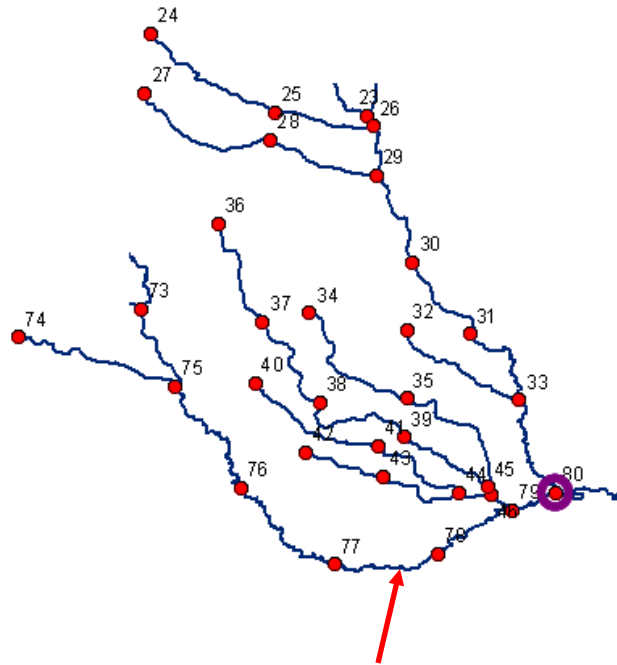
Spill_number	9																							
Spill_between	75																							76
FS	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	
	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	
	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	
	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	
	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	
	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	
	33	33	34	34	34	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	
	34	34	35	35	36	37	37	38	38	39	39	40	40	41	41	41	41	41	41	41	41	41	41	
	35	35	36	36	37	38	38	39	39	40	40	41	41	42	42	42	42	42	42	42	42	42	42	
	36	36	37	37	38	39	39	40	40	41	41	42	42	43	43	43	43	43	43	43	43	43	43	
	37	37	38	38	39	40	40	41	41	42	42	43	43	44	44	44	44	44	44	44	44	44	44	
	38	38	39	39	40	41	41	42	42	43	43	44	44	45	45	45	45	45	45	45	45	45	45	
	39	39	40	40	41	42	42	43	43	44	44	45	45	46	46	46	46	46	46	46	46	46	46	
	40	40	41	41	42	42	43	43	43	44	44	45	45	46	46	46	46	46	46	46	46	46	46	
	41	41	42	42	43	44	44	45	45	46	46	47	47	47	47	47	47	47	47	47	47	47	47	
	42	42	43	43	44	45	45	46	46	47	47	48	48	48	48	48	48	48	48	48	48	48	48	
	43	43	44	44	45	46	46	47	47	48	48	49	49	49	49	49	49	49	49	49	49	49	49	
	44	44	45	45	46	47	47	48	48	49	49	50	50	50	50	50	50	50	50	50	50	50	50	
	45	45	46	46	47	48	48	49	49	50	50	51	51	51	51	51	51	51	51	51	51	51	51	
	46	46	47	47	48	49	49	50	50	51	51	52	52	52	52	52	52	52	52	52	52	52	52	
	47	47	48	48	49	50	50	51	51	52	52	53	53	53	53	53	53	53	53	53	53	53	53	
	48	48	49	49	50	51	51	52	52	53	53	54	54	54	54	54	54	54	54	54	54	54	54	
	49	49	50	50	51	52	52	53	53	54	54	55	55	55	55	55	55	55	55	55	55	55	55	
	50	50	51	51	52	53	53	54	54	55	55	56	56	56	56	56	56	56	56	56	56	56	56	
	51	51	52	52	53	54	54	55	55	56	56	57	57	57	57	57	57	57	57	57	57	57	57	
	52	52	53	53	54	55	55	56	56	57	57	58	58	58	58	58	58	58	58	58	58	58	58	
	53	53	54	54	55	56	56	57	57	58	58	59	59	59	59	59	59	59	59	59	59	59	59	
	54	54	55	55	56	57	57	58	58	59	59	60	60	60	60	60	60	60	60	60	60	60	60	
	55	55	56	56	57	58	58	59	59	60	60	61	61	61	61	61	61	61	61	61	61	61	61	
	56	56	57	57	58	59	59	60	60	61	61	62	62	62	62	62	62	62	62	62	62	62	62	
	57	57	58	58	59	60	60	61	61	62	62	63	63	63	63	63	63	63	63	63	63	63	63	
	58	58	59	59	60	61	61	62	62	63	63	64	64	64	64	64	64	64	64	64	64	64	64	
	59	59	60	60	61	62	62	63	63	64	64	65	65	65	65	65	65	65	65	65	65	65	65	
	60	60	61	61	62	63	63	64	64	65	65	66	66	66	66	66	66	66	66	66	66	66	66	
	61	61	62	62	63	64	64	65	65	66	66	67	67	67	67	67	67	67	67	67	67	67	67	
	62	62	63	63	64	65	65	66	66	67	67	68	68	68	68	68	68	68	68	68	68	68	68	
	63	63	64	64	65	66	66	67	67	68	68	69	69	69	69	69	69	69	69	69	69	69	69	
	64	64	65	65	66	67	67	68	68	69	69	70	70	70	70	70	70	70	70	70	70	70	70	
	65	65	66	66	67	68	68	69	69	70	70	71	71	71	71	71	71	71	71	71	71	71	71	
	66	66	67	67	68	69	69	70	70	71	71	72	72	72	72	72	72	72	72	72	72	72	72	
	67	67	68	68	69	70	70	71	71	72	72	73	73	73	73	73	73	73	73	73	73	73	73	
	68	68	69	69	70	71	71	72	72	73	73	74	74	74	74	74	74	74	74	74	74	74	74	
	69	69	70	70	71	72	72	73	73	74	74	75	75	75	75	75	75	75	75	75	75	75	75	
	70	70	71	71	72	73	73	74	74	75	75	76	76	76	76	76	76	76	76	76	76	76	76	
	71	71	72	72	73	74	74	75	75	76	76	77	77	77	77	77	77	77	77	77	77	77	77	
	72	72	73	73	74	75	75	76	76	77	77	78	78	78	78	78	78	78	78	78	78	78	78	
	73	73	74	74	75	76	76	77	77	78	78	79	79	79	79	79	79	79	79	79	79	79	79	
	74	74	75	75	76	77	77	78	78	79	79	80	80	80	80	80	80	80	80	80	80	80	80	
	75	75	76	76	77	78	78	79	79	80	80													
	76	76	77	77	78	79	79	80	80															
	77	77	78	78	79	80	80																	
	78	78	79	79	80																			
	79																							
	80																							

Spill_number	9		
Spill_between	75		76
FS	P	P	P
Hr	1	0.93	0.8
	29	29	30
	30	30	46
	44	46	75
	46	75	76
	75	76	
	76		

Result	for	spill	numbe	9	which	is	betwe	75	and	76
	J	C	MM	Pe	Avg					
		29	0.143	0.15	0.046	0.113				
		30	0.402	0.28	0.308	0.331				
		46	0.125	0.25	0.192	0.188				
		75	0.33	0.3	0.071	0.234	←			
		76	0	0.02	0.383	0.134	←			

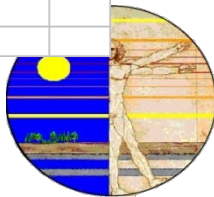


Results



Spill_number	10																		
Spill_between	77	78																	
FS	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
Hr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	27	29	29	30	30	30	30	30	30	31	31	31	31	31	31	31	31	31	31
	28	30	30	31	31	31	31	31	31	33	33	33	33	33	33	33	33	33	77
	29	31	31	33	33	33	33	33	33	46	46	46	46	46	46	46	46	46	77
	30	33	33	41	41	43	44	44	44	46	46	46	46	46	46	46	46	46	78
	31	41	41	43	43	44	46	46	46	76	76	76	76	76	76	76	76	76	78
	32	42	43	44	44	46	76	76	76	77	77	77	77	77	77	77	77	77	78
	33	43	44	46	46	76	77	77	77	78	78	78	78	78	78	78	78	78	78
	34	44	46	75	76	77	78	78	78										
	35	46	75	76	77	78	80	80	80										
	36	75	76	77	78	80													
	37	76	77	78	80														
	38	77	78	80															
	39	78	80																
	40	80																	
	41																		
	42																		
	43																		
	44																		
	45																		
	46																		
	74																		
	75																		
	76																		
	77																		
	78																		
	79																		
	80																		

Result	for	spill	number	10	which	is	betwe	77	and	78									
	J	C	MM	Pe	Avg														
	31	0.954	0.93	0.699	0.86														
	77	0.046	0.07	0.301	0.14	←													



Conclusions

- Modified ASFS algorithm performed well for both regions tested by total of 20 spill events.
 - The correct answer was kept in final sets for all 20 spill events.
 - The worst performance was reducing initial class set of 26 elements to a final selected set of 7 elements.
 - The best performance was reducing the initial class set of 27 elements to a final selected set of 2 elements.
- Final set is ranked according to the average of posterior conditional probabilities obtained for each single feature and joint behavior of the two.
- Once the statistical data is obtained from simulations (takes about 2 weeks), this method is fast (takes about 30 seconds to analyze all 20 spills).





Thank you...

